Physics 451 - Fall 2012

Homework #22
Due Thursday, Nov 29, by 7pm

Please place your assignment in the “Physics 451” slot across from N373 ESC.
Help sessions T Th: from 3pm to 6pm – room 337 ESC

List of problems (from the textbook):

5.1
5.2
5.4
5.6

Hints: Problem 5.1: a) The old coordinates are \( \vec{r}_1 = (x_1, y_1, z_1) \) and \( \vec{r}_2 = (x_2, y_2, z_2) \). The new coordinates are \( \vec{R} = (X, Y, Z) \) and \( \vec{r} = (x, y, z) \). To express the “Del” operator in new coordinates, find an expression for each of the components along x, y, z, and use partial derivatives. For example, use: \( \nabla_{1,x} = \frac{\partial}{\partial x_1} = \frac{\partial X}{\partial x_1} \frac{\partial}{\partial X} + \frac{\partial x}{\partial x_1} \frac{\partial}{\partial x} \)
b) Express the Hamiltonian in terms of old coordinates first and transform its expression in terms in new coordinates.
c) Divide the Schrödinger equation by \( \psi \frac{\partial^2}{\partial r^2} \psi \) and separate a term that depends on \( r \) only from a term that depends on \( R \) only.

Problem 5.2: numerical application
a) Hydrogen atom (1 proton, 1 electron): \( m_p = 1.67 \times 10^{-27} \text{kg} \); \( m_e = 9.1 \times 10^{-31} \text{kg} \)
b) Deuterium atom (1 proton + 1 neutron, 1 electron): \( n = m_p \) so \( m_{\text{nucleus}} = 2m_p \)
c) Positronium atom (1 positron, 1 electron): \( m_{\text{pos}} = m_e \)
d) Muonic hydrogen (1 proton, 1 muon): \( m_{\text{muon}} = 206.77m_e \)

Problem 5.6: the stationary states of the infinite square well are: \( \psi_n(x) = \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi x}{a} \right) \)

To find \( \langle (\Delta x)^2 \rangle \) in the three cases, you will basically need to calculate:

\( \langle x \rangle_n, \langle x^2 \rangle_n \) and \( \langle x \rangle_{sl} \) (by integration – between limits 0 and a)

\( \int x^2 \sin^2(\alpha x) dx = \left[ \frac{x^3}{6} - \frac{x \cos(2\alpha x)}{4\alpha^2} - \left( \frac{x^2}{4\alpha} - \frac{1}{8\alpha^3} \right) \sin(2\alpha x) \right] \)

\( \int x \sin(\alpha x) \sin(\beta x) dx = \frac{1}{2} \left[ \frac{\cos[(\alpha - \beta)x]}{(\alpha - \beta)^2} - \frac{\cos[(\alpha + \beta)x]}{(\alpha + \beta)^2} + x \frac{\sin[(\alpha - \beta)x]}{(\alpha - \beta)} - x \frac{\sin[(\alpha + \beta)x]}{(\alpha + \beta)} \right] \)