Physics 451- Fall 2012

Homework #22

Due Thursday, Nov 29, by 7pm

Please place your assignment in the "Physics 451" slot across from N373 ESC.
Help sessions T Th: from 3pm to 6pm – room 337 ESC

List of problems (from the textbook):

5.1
5.2
5.4
5.6

Hints: Problem **5.1**: a) The old coordinates are $\vec{r}_1 = (x_1, y_1, z_1)$ and $\vec{r}_2 = (x_2, y_2, z_2)$. The new coordinates are $\vec{R} = (X, Y, Z)$ and $\vec{r} = (x, y, z)$. To express the "Del" operator in new coordinates, find an expression for each of the components along x,y,z, and use partial derivatives. For example, use: $\nabla_{1,x} = \frac{\partial}{\partial x_1} = \frac{\partial X}{\partial x_1}\frac{\partial}{\partial X} + \frac{\partial x}{\partial x_1}\frac{\partial}{\partial x}$

b) Express the Hamiltonian in terms of old coordinates first and transform its expression in terms in new coordinates.

c) Divide the Schrödinger equation by $\psi_r \psi_R$ and separate a term that depends on *r* only from a term that depends on *R* only.

Problem **5.2**: numerical application

a) Hydrogen atom (1 proton, 1 electron): $m_p = 1.67\times10^{-27} kg$; $m_e = 9.1\times10^{-31} kg$
b) Deuterium atom (1 proton + 1neutron, 1 electron): $m_n = m_p$ so $m_{nucleus} = 2m_p$
c) Positronium atom (1 positron, 1 electron): $m_{pos} = m_e$
d) Muonic hydrogen (1 proton, 1 muon): $m_{muon} = 206.77 m_e$

Problem **5.6:** the stationary states of the infinite square well are: $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$

To find $\left\langle (\Delta x)^2 \right\rangle$ in the three cases, you will basically need to calculate:

$\langle x \rangle_n$, $\langle x^2 \rangle_n$ and $\langle x \rangle_{nl}$ (by integration – between limits 0 and a)

$$\int x^2 \sin^2(\alpha x)dx = \left[\frac{x^3}{6} - \frac{x\cos(2\alpha x)}{4\alpha^2} - \left(\frac{x^2}{4\alpha} - \frac{1}{8\alpha^3}\right)\sin(2\alpha x)\right]$$

$$\int x \sin(\alpha x)\sin(\beta x)dx = \frac{1}{2}\left[\frac{\cos\left[(\alpha-\beta)x\right]}{(\alpha-\beta)^2} - \frac{\cos\left[(\alpha+\beta)x\right]}{(\alpha+\beta)^2} + x\frac{\sin\left[(\alpha-\beta)x\right]}{(\alpha-\beta)} - x\frac{\sin\left[(\alpha+\beta)x\right]}{(\alpha+\beta)}\right]$$