Hints:

For problem 4.18: The point where to start is to calculate the norm (or the norm square) of the vector $L_z f_l^m$ in order to determine the expression for $A_l^m$ (or its square). By doing so, you will introduce the product $L_z L_z$ which you can replace by its expression in terms of $L^2$ and $L_z$ (using equation 4.112). Use the fact that $f_l^m$ are normalized.

For problem 4.21, use a test function in order to apply all the successive derivations.

Trigonometric relationships: \[ \frac{d}{d\theta} \cot \theta = -\csc^2 \theta = -\frac{1}{\sin^2 \theta} \] and \[ \csc^2 \theta = 1 + \cot^2 \theta \]

Useful integral for problem 4.22

\[ \int_0^{\pi} \sin^{2l+1} \theta d\theta = \frac{2 \cdot 4 \cdot 6 \ldots (2l)}{1 \cdot 3 \cdot 5 \ldots (2l+1)} = \frac{(2^l)!}{(2l+1)!} \]

For question c), remember the integral for angular normalization: \[ \int_0^{2\pi} \int_0^{\pi} |Y_l^m|^2 \sin \theta d\theta d\phi \]