Physics 145: Mechanical & Acoustical Resonators

What's the point? Generalize the concept of AC impedance to sinusoidally-driven mechanical and acoustical systems.

Equipment: (1) Magnetically-driven mass-spring oscillator system with VI control, (2) Helmholtz oscillator (small speaker, Wavetek signal generator Christmas bulb, lump of clay, computer microphone).

Introduction

The damped mass-spring oscillator exhibits a well-defined mechanical resonance at $\omega_0 = \sqrt{k/m}$, where k is the Hooke's-law spring constant and m is the mass. Energy is traded back and forth during each oscillation between kinetic and potential energy. The frictional force on the moving mass is proportional to its velocity via the damping coefficient b: $F_{\text{friction}} = -bv$. These parameters have a close analogy to the RLC circuit case, where $m \to L$, $k \to 1/C$, $b \to R$, $F \to V$, and $v \to I$. After interchanging these variables, all of the impedance-related concepts and equations above can be used to describe the details of a mechanical resonance. The velocity response, for example, is: $|v_0| = \dfrac{|F_0|}{\sqrt{b^2 + (\omega m - k/\omega)^2}}$.

The Helmholz resonator is an especially simple example of acoustical resonance. This is generally a cavity with rigid walls and a relatively small tube-like opening to the outside. A speaker driver or other sound source just outside the opening excites the resonance inside the cavity. By analogy to the mass-spring oscillator, the air inside the tube-shaped opening is the mass, the compressible gas inside the cavity is the spring, and viscous friction between the air and the cavity walls provides some damping. With a little work, one can roughly approximate the resonance frequency with the equation $f_0 = (c/2\pi)\sqrt{A/VL'}$, where A and L' are the respective cross-sectional area and effective length of the tube-shaped opening, V is the internal volume of the resonator cavity, and c is the speed of sound in air (343 m/s). The effective length of the opening is somewhat longer than the actual length due to the fact that some of the air just inside and just outside the opening is also moving. A good approximation for the effective length is $L' = L + 1.45R$, where L and R are the actual length and radius of the opening. You might take a few minutes to follow the discussion and derivation at http://www.phys.unsw.edu.au/jw/Helmholtz.html.

PROCEDURE

A: Measure the frequency response of a damped mass-spring oscillator.

1) Working with a team of up to three students, familiarize yourself with the damped mass-spring resonator apparatus. Use either of two available stations. Analog output channel 1 on the interface box drives a current through the large electromagnet, which creates the magnetic force that drives the mass-spring system. Analog input channel 2 (called ai1 by Labview) reads in the voltage from the smaller coil, which is proportional to the velocity of the mass.
2) Given the mass, the spring constant, and the damping coefficient of your apparatus, predict the resonance frequency of your oscillator. The combined mass m (of the rod, magnet, and damper) is printed on the front of the apparatus. The spring constant k can be approximated by resting a 500 gram mass on the damper and observing the resulting displacement. The approximate damping coefficient b should be printed on the underside of the damper disc. Enter the resulting b, m, and k values into your lab notebook. Use *impedance.nb* to plot the expected "response" curve vs frequency.
3) Use the HarmonicOscillator VI to map out the frequency-dependent velocity of the oscillator over a reasonable frequency range that includes both the low and high-frequency tails of the resonance. Collect about 30 data points, manually recording the peak-to-peak signal amplitude at each point. Choose an an appropriate linear frequency range that includes the interesting features and which approximately centers the resonance in the middle of range. Enter the data into Excel and plot it for your lab notebook. Show how the measured resonance frequency compares to your predicted value? They should be quite close.
4) For one-point extra credit, use either LoggerPro or Mathematica (NLSQ.nb) to fit a resonance curve to your resonance . In contrast to the approach taken with the electrical LRC circuit, fix the mass at the printed value, and refine both b and k, along with the scale factor (A). Make sure that your result is reasonable.

B: Measure the frequency response of an acoustical Helmholtz resonator.

1) Choose a Helmholtz resonator cavity from among the objects provided. Excite the resonance by blowing a gentle stream of air across the end of the opening. With a little practice, you may be able to produce a clean musical note, though you shouldn't expect to outshine Fritz Richard after only one day.
2) Mount a small acoustic driver (i.e. speaker) a short distance away from the mouth of the resonator and drive it with a sinusoidal waveform from the Wavetek signal generator. Search the frequency range from 100 Hz to 600 Hz to locate the Helmholtz resonance. When you find the resonance, the audible sound will get much louder as speaker approaches the cavity. Once you find the resonance, adjust the distance between the driver and the resonator until you maximize the response.
3) Use the expression in the introduction to predict the resonance frequency. How does your prediction compare to the observed resonance frequency? If there is a discrepancy, can you explain it?

4) Measure the acoustic response by dangling a small microphone into the center of the cavity by its leads, and then plug it in to the microphone jack (pink) in the back of your computer. Use rods and clamps to fix the speaker in an optimal position. Create a labview VI that reads in a 0.10 second sample from the microphone at a 20 kHz sample rate, and sends the output into a Graph. Use the **Express → Input → Sound Input** VI, and create numeric controls to set the sample rate and the sample duration. Place these items inside of a While loop, and set it to run continuously. In the frequency range below 1 kHz, the graph should show a clean sinuousoidal signal coming in from your mic that gets updated several times per second.
5) Map out the frequency response of the resonator over a reasonable frequency range that includes both the low and high-frequency tails of the resonance. Collect about 30 data points, manually recording the peak-to-peak signal amplitude at each point. Include amplitude error estimates. Record your data in an Excel spreadsheet and plot the results on a linear horizontal scale for your lab notebook. Note that you don't need to attempt a quantitative fit to the data, though you can if you want to.