Physics 145: LRC circuits

What's the point? Expand your understanding of AC gain and impedance by building and characterizing a sterotypical LRC circuit.

Equipment: Parts I: LRC component board, LC meter, signal generator/oscilloscope stack, frequency meter, amplifier, cables.

Introduction:

The series LRC oscillator (see Figure 1) is an important case that has myriad applications. Driven by an AC voltage source, the oscillating current carries energy back and forth between the capacitor (electric field energy) and the inductor (magnetic field energy). In series, the total impedance of these three elements is $Z = Z_R + Z_L + Z_C = R + i(\omega L - 1/\omega C)$, which has magnitude $|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$ and phase $\tan(\phi) = (\omega L - 1/\omega C)/R$. For a given driving voltage, the magnitude of the current oscillations will be

$$I_0 = \left|\frac{V_0}{Z}\right| = \frac{|V_0|}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}},$$

which is maximized by minimizing |Z| (see Figure 2). To minimize |Z|, select a driving frequency that sends the total reactance $\omega L - 1/\omega C$, to zero. This special frequency, $\omega_0 = 1/\sqrt{LC}$, is called the resonance frequency of the circuit. In the absence of a resistance *R*, the total impedance would go to zero at $\omega = \omega_0$, and the current amplitude would either run away to infinity or else the circuit would fail in a spectacular display of light and heat. The resistor damps the oscillations by dissipating energy, so that the energy pumped into the circuit by the AC source doesn't accumulate without limits.

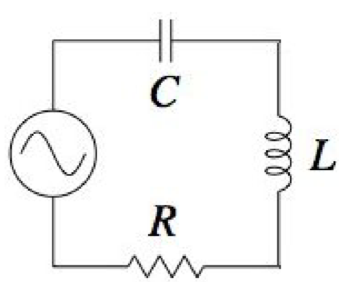

Figure 1. Driven series LRC circuit. The symbol at the left of the circuit is the AC voltage source that drives the circuit.

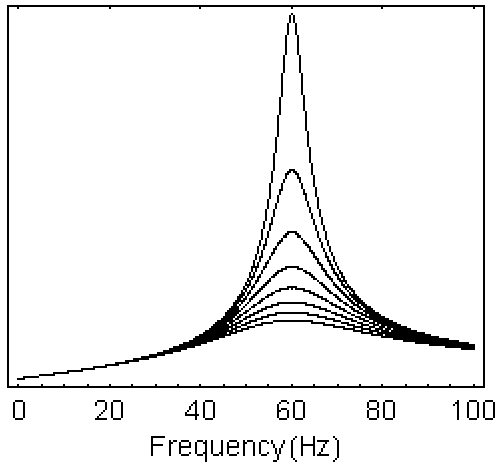

Figure 2. Current-amplitude curves for a damped-driven LRC oscillator (L = 0.265 mH, C = 0.265 mF) for several different values of damping resistor R.

The magnitude and phase of the gain of an LRC circuit are respectively

$$|G| = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \text{ and } \tan(\phi_G) = \frac{-(\omega L - 1/\omega C)}{R}.$$

At resonance, the gain has a magnitude of 1 and phase of zero. At all other frequencies, the magnitude of the gain is less than 1.

The reactive components (L and C) in an LRC network do not dissipate energy. Only reactive components can store energy, and only resistive elements can dissipate energy. The rate of energy dissipation has units of power, and can be easily calculated, as a time-average quantity, in terms of the magnitude and phase of the impedance: $Z = |Z|e^{i\phi}$. Here we list several distinct but equivalent expressions for average power, any one of which might be the most convenient depending which circuit parameters are known.

$$\overline{P} \equiv I_{rms}^2 R = I_{rms}^2 |Z| \cos(\phi) = I_{rms} V_{rms} \cos(\phi) = \frac{V_{rms}^2}{|Z|} \cos(\phi) = \frac{V_{rms}^2}{R} \cos^2(\phi)$$

Here, we defined the rms current and voltage in the standard way: $V_{rms} \equiv |V_0| / \sqrt{2}$, $I_{rms} \equiv I_0 / \sqrt{2}$, and observed that $R = |Z|\cos(\phi)$.

Many physical systems exhibit resonance effects. At the time of this writing, the power supply fan in the back of the author's desktop computer has begun to come loose from it's mount The loud noise that it is making suggests that the rotational driving frequency of the fan has excited a vibrational resonance in the structure that supports it. The crystal shattered by an opera singer, the tall building that sways back and forth during an earthquate, the rattling of a car driving over the top of the "wake-up" grooves cut into the shoulder of an interstate highway -- these are all examples of driven damped resonance, though a shattering crystal or a collapsing building could clearly use some more damping to limit the energy accumulation.

PROCEDURE

A: Measure and analyze the frequency response of an LRC resonator.

For the LRC resonator circuit shown above in the introduction, follow the same procedure that you used for the high-pass filter lab to acquire and analyzing the frequency response curve. You want your resonance peak somewhere between 2.0 and 6.0 kHz. When you get to the part of the exercise that involves curve fitting, employ a model of the form $|G| = \dfrac{A}{\sqrt{1+[(2\pi f L/R) - 1/(2\pi f RC)]^2}}$, using A and L and C as variables (fix R to its measured value). Skip the part of the exercise that pertains to the f_{3db} frequency, which is only relevant to high and low-pass filters, but still compare the measured and fitted values of L and C.

Tips: (1) Before you start collecting data, quickly scan the whole frequency range of your oscilloscope to verify that you have an acceptable resonance peak (not to broad, not too narrow). (2) You may need to try several different resistors to get an acceptable width.

B: Compute the resonance frequency

Using the fitted values of L and C, compute the expected resonance frequency in Hz: $f_{res} = \dfrac{1}{2\pi\sqrt{LC}}$. Compare it to the location of the observed resonance peak. Comment on the effect of R on the width and location of the resonance peak.