Physics 145: FOURIER SPECTRA

What's the point? You will become acquainted with the use of the Fourier transform to a convert temporal signal into a frequency signal (i.e. frequency spectrum). You will then measure and analyze the Fourier spectra of a variety of sound signals that you will produce in the lab.

Equipment: Computer equipped with computer microphone and headphones, tuning fork set, balloons to pop.

Introduction: While the uninitiated may feel more comfortable viewing a time-varying signal in the **time domain**, scientists and engineers often prefer to transform a signal into the **frequency domain** for detailed analysis. As you work through this lab, you will hopefully gain some appreciation for this approach. You will be introduced to **Fourier analysis**, a powerful method of analyzing oscillatory signals, such as sound waves.

You should already understand that a **function** is an operation that takes one value as input and returns another value as output. The function $f(x)$, for example, indicates a function that takes x and returns f. Here, we introduce the concept of a **functional transform**, which by definition, takes one function as input, and returns another function as output. The notation $\Im_y[f(x)] = g(y)$ indicates a transform that takes function $f(x)$ as input and returns $g(y)$ as output.

One of the most common transforms used for time-signal analysis is the **Fourier transform**:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt\,,$$

which transforms a time-domain signal $f(t)$ into its conjugate **frequency spectrum** $F(\omega)$, where $\omega = 2\pi\nu$ is known as angular frequency. While cyclic frequency ν is usually measured in Hz, angular frequency is measure in radians/second. The **inverse Fourier transform**:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega\,,$$

does just the opposite, taking $F(\omega)$ back into $f(t)$. Either function, $f(t)$ or $F(\omega)$, contain the same information about the time signal, though the information is represented quite differently in the two domains. Furthermore, while the time-domain signal $f(t)$ is generally a real-valued function, $F(\omega)$ can be a complex-valued function.

To understand the frequency domain, one should first know that most time-domain signals can be viewed an infinite superposition of sinusoidal waves, each having a distinct frequency. The frequency-domain spectrum is the function that reveals the amplitude and phase of the contribution from each available frequency. If you have ever used a graphic equalizer (see figure 1), you are already be somewhat familiar with frequency spectra. Each vertical column of the equalizer represents a narrow frequency range, and shows a bar height that indicates the magnitude of the contribution from that frequency range. Because the bar height actually represents $|F(\omega)|^2$, we call such a graph a frequency **power spectrum**. Though they lack phase information, power spectra are easy to interpret and contain a great deal of information about their respective time signals.

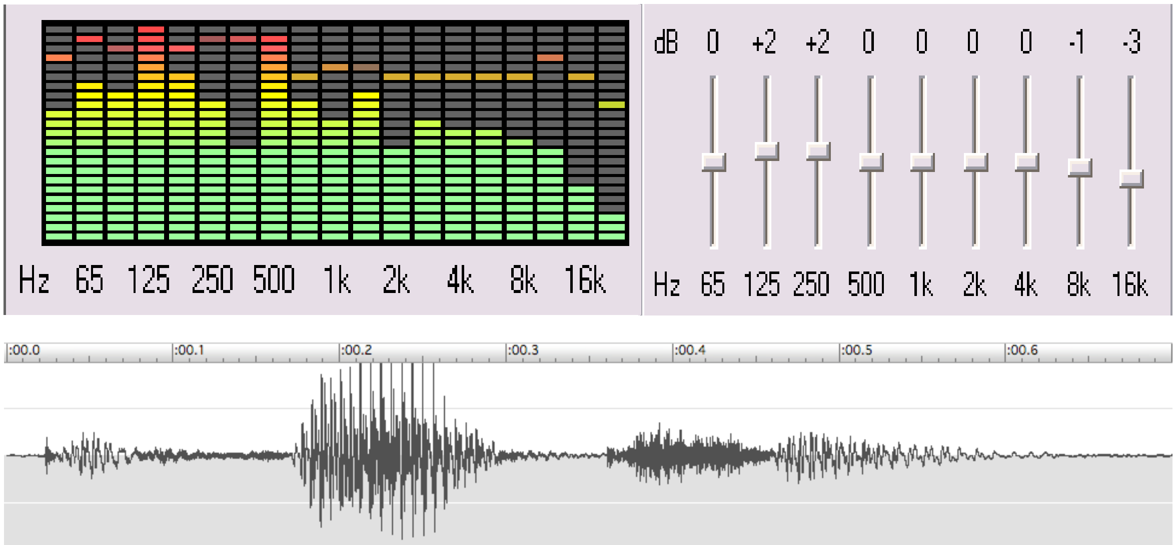

Figure 1. The graphic equalizer (top) shows the Fourier power spectrum of a time-domain signal (bottom).

An arbitrary time signal $f(t)$ will have a continuous frequency spectrum $F(\omega)$. The Fourier transform of a Gaussian peak, for example, is another Gaussian. Periodic time signals, on the other hand, are *special* in that they have **discrete frequency spectra**. You would do well to repeat that last sentence three times for emphasis -- this is a very important concept. Rather than having a continuous range of frequency contributions, only a discrete set of frequencies (integer multiples of some **fundamental frequency** $f_0 = \omega_0/2\pi$) are allowed to have a non-zero amplitude. Thus, the power spectrum of a periodic function tends to look like an evenly-spaced comb of narrow spikes, each spike (or **harmonic**) having a different intensity (see figure 2).

A pure musical note, regardless of the voice or instrument, is a periodic time signal. The square-wave and saw-tooth waveforms that you have seen produced by an electronic signal generator are periodic. Many other physical systems also oscillate in a periodic fashion. Note that periodic does not mean sinusoidal. It simply means that the signal repeats itself in time every so often. For a truly sinusoidal signal with frequency ω_0, the power spectrum only contains one peak at f_0. But for a more complex periodic function, many harmonics of the fundamental frequency (e.g. f_0, $2f_0$, $3f_0$, $4f_0$, $5f_0$, ...) will have non-zero amplitudes.

Any periodic function can also be expanded as a discrete series of sine and cosine terms, each

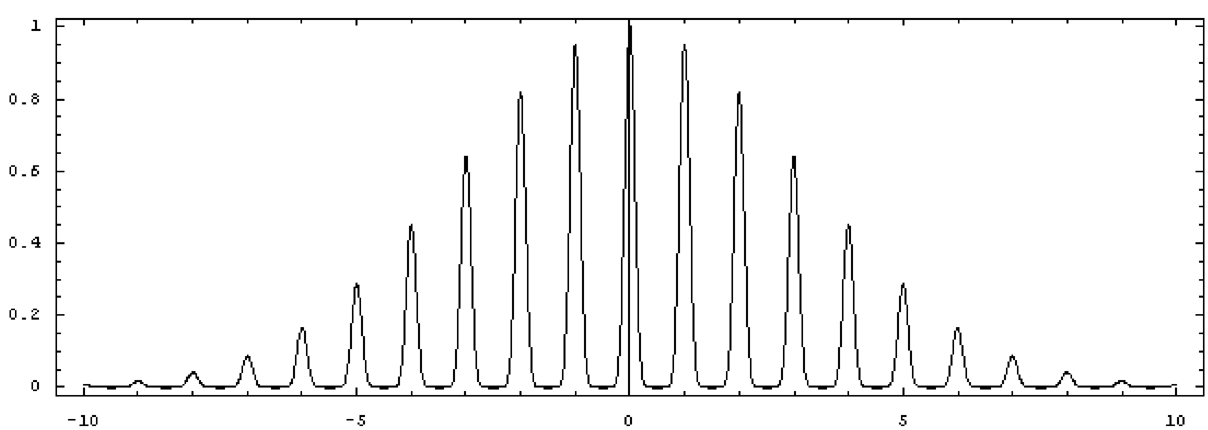

Figure 2. Fourier power spectrum of a signal with fundamental frequency $f_0 = 1$ kHz.

term representing one of the harmonics of the fundamental frequency ω_0:

$$f(t) = A_0 + A_1\cos(\omega_0 t) + B_1\sin(\omega_0 t) + A_2\cos(2\,\omega_0 t) + B_2\sin(2\,\omega_0 t) + A_3\cos(3\,\omega_0 t) + B_3\sin(3\,\omega_0 t) + \ldots$$

Somewhat analogous to a Taylor series, this type of expansion is called a **Fourier series**. **Fourier analysis** is simply is the calculation of the coefficients in the Fourier series expansion of a signal. The intensity a specific harmonic in the power spectrum will be $I_n = A_n^2 + B_n^2$. As an example, the periodic function $\sin^2(2\pi x)$ has Fourier series coefficients:

$$A_0 = 0.5,\ A_1 = 0,\ A_2 = -0.5,\ A_{n>2} = 0, \text{ and } B_n = 0,$$

so that it's power spectrum coefficients are:

$$I_0 = 0.25,\ I_1 = 0,\ I_2 = 0.25, \text{ and } I_{n>2} = 0.$$

Another interesting example is sawtooth function $s(x)$ with peak amplitudes of ±1, which has Fourier series coefficients:

$$A_0 = 0,\ A_1 = 0.8106,\ A_2 = 0,\ A_3 = 0.0901,\ A_4 = 0,\ A_5 = 0.0324, \ldots, \text{ and } B_n = 0,$$

so that it's power spectrum coefficients are:

$$I_0 = 0,\ I_1 = 0.6570,\ I_2 = 0,\ I_3 = 0.0081,\ I_4 = 0,\ I_5 = 0.0011 \ldots$$

PROCEDURE

A: Become familiar with the Labview VI for generating Fourier spectra.

1) Copy the Fourier_waveform.vi to your own working area. Open it up and explore its front panel and block diagram. Use the VI to explore the Fourier power spectra several computer-generated waveforms. Develop some intuition as to how the variable waveform parameters affect the spectra. Include and describe a few interesting results in your lab notebook.
2) Learn how to switch between linear and logarithmic scales on the X and Y axes. Ask your TA to show you how to use the Graph Palette to zoom in on features of interest.

B: Generate and analyze Fourier spectra of sound-wave samples.

1) Copy the Fourier_sound.vi to your own working area. Open it up and explore its front panel and block diagram. Turn off the *Enable Filter* switch and set the *Card/Interface* switch to "Sound Card" on the VI front panel. Set the sample rate at 20 kHz, and enter the number of samples that will yield a one-second sound sample. Plug the headset microphone into the microphone jack (pink) at the back of your computer, and the headset speaker in the speaker jack (green). Collect a few samples of yourself whistling a steady note, and observe the relevant peaks in the resulting power spectrum. You should be able to hear the playback of your sound sample in the headphones. Ignore the filtered time and frequency-domain graphs at the right-hand side of the front panel.
2) Now record the note produced by one of the tuning forks provided and generate its Fourier power spectrum. Use the spectrum to accurately determine the fundamental vibration frequency of the fork (a log scale or zoom view may help). Try two more forks. How well do the measured frequencies agree with the nominal values printed on the tuning forks themselves? Print a few time and frequency graphs for your lab notebook along with your observations and explanation.
3) The sample rate that you choose is dictated in part by the maximum frequency that you want to observe in your spectrum - the sample rate must be twice the maximum frequency. This is called the **Nyquist sampling criterion**. For example, if you want to measure frequencies up to 10 kHz, you need to collect at least 20,000 samples/second. To see this, choose a tuning

fork with a fundamental frequency f_0 somewhere near 1 kHz. For convenience, use the graph palette to zoom in on a time region containing about 10 fundamental oscillations and turn the *x*-axis autoscaling off. Start the sample rate at a value equal to roughly $4f_0$, using a time duration of 1 second. Then lower the sample frequency to $3f_0$, and then to $2f_0$, and finally to f_0. What happens to your time and frequency-domain signals when the Nyquist frequency (i.e. half the sample rate) drops below the frequency of the sound that you are measuring? Print a few informative graphs for your lab notebook and explain what you see there.

4) Generate a Fourier power spectrum for a few abrupt sounds such as a clapping, yelling, or popping a balloon. Adjust the sample frequency and sample time if necessary. Once again, record your results and observations in your lab notebook.

C: Use Fourier voice spectra for vowel-sound recognition.

1) Generate Fourier spectra for each of the following vowel sounds: ā, ē, ō, ū (pronounce as "ooh" not "you"). You may need practice a few times before obtaining good spectra. Note that the cleanest spectra generally come from notes that are sung rather than spoken. It is also important to hold the note steady during the entire sampling interval.
2) Print each spectrum and label it lightly on the back, but not on the front. Each lab partner should generate their own spectra. Now see if you can identify the vowels associated with each of your lab partner's (or someone else's) spectral printouts without looking at the labels on the back. Then save your spectra in your lab notebook along with your observations. Discuss which harmonics tend to be emphasized for each vowel sound.

D: Apply frequency filters to vocal input.

1) Increase your sampling interval to 2 or more seconds (at a 20 kHz sampling rate). Record vocal input with the microphone, and verify that you can hear the playback clearly in the headphones. On the front panel, turn the *Filter Enable* switch to *On*. You should now see (and hear) that a Butterworth band-pass filter has been applied to your input signals. The unfiltered signals appear in the graphs on the left-hand side of the front panel, while the filtered signals appear on the right-hand side.
2) Try a variety of filter cutoff settings and observe the effects of the filter on both time and frequency-domain signals. Print graphs that illustrate a couple of interesting cases for your lab notebook, and include observations and explanation.