Physics 145: Time and frequency responses of low-pass filters

What's the point? Apply the concept of AC impedance to the frequency response of a low-pass filter, and explore the relationship between time and frequency responses.

Equipment: LRC component board, LC meter, signal generator/oscilloscope stack, frequency meter, amplifier, cables.

INTRODUCTION

A *time-response* study addresses the significant question, "How does the output vary with time when the input experiences an instantaneous change?" We can be fairly certain that a real transducer will not respond instantaneously. Even a device that responds very quickly will take a finite amount of time to complete the change. This is because real devices need time to convert energy from one form into another. We define the *rise time* τ to be time required for the output to complete about (1-1/*e*) = 63% of the full response to a sudden increase in the input. Similarly, we define the *fall time* τ to be time required for the output to complete 63% of the full response to a sudden decrease in the input. The rise and fall times can be quite different for a given device. Fig. 5 illustrates the time variation of a typical transducer output following a step change at the input. Because a square wave alternately increases and decreases sharply, it allows one to observe both the rise time and the fall time of one device with a single experiment.

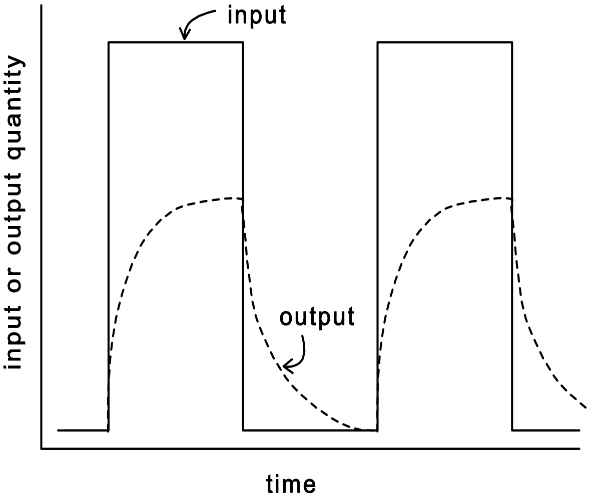


Figure 1. Illustration of a typical time response to instantaneous input changes.

Those acquainted with Fourier analysis will realize that the time response functions (TRFs) and the frequency response functions (FRFs) have a reciprocal relationship. If we want to be mathematically rigorous, the time-response to a delta-function input spike is the inverse Fourier transform of the frequency response of the device. More intuitively, they have a type of inverse relationship. If a device has a good response at high frequencies, it will respond quickly to sudden changes in time (i.e. large frequencies correspond to short times). If it has a good response at zero frequency, it will be capable of a DC output in time (i.e. small frequencies correspond to long times). Thus, a short rise time implies a large high-frequency cutoff; a long rise time implies a small high-frequency cutoff. And in general, one can use the frequency response of a device to predict its time response to an arbitrary non-sinusoidal input (even a square wave). Ultimately, the TRF and the FRF are just two different ways at looking at the same system, since they contain the same information.

The low-pass filter is the stereotypical system for understanding the relationship between time and frequency response measurements. The frequency-dependent gain of a low-pass filter has the form $|G| = \frac{1}{\sqrt{1+(f/f_{3db})^2}}$, where the response time (τ.) and the cutoff frequency ($f_{3\mathrm{db}}$) are related by a simple inverse: $f_{3db} = 1/(2\pi\,\tau)$.

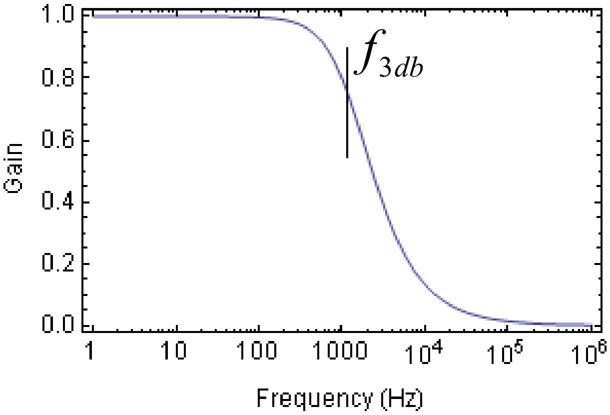
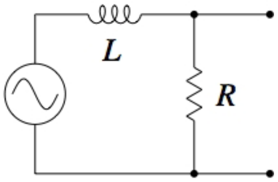


Figure 2. Frequency-response function of a low-pass filter with $f_{3\mathrm{db}}$ = 1 kHz.

PROCEDURE

A: Measure and analyze the frequency response of a low-pass RL filter.



For the RL circuit shown at the right, follow the same procedure that you used for the high-pass filter lab to acquire and analyze the frequency response curve. When you get to the part of the exercise that involves curve fitting, employ a model of the form $|G| = \frac{A}{\sqrt{1+(2\pi f\, L/R)^2}}$, with A and L as variables (fix R to its measured value). We anticipate that this exercise will proceed smoothly and quickly based on your previous experience with high-pass filters.

B: Measure the time response of a low-pass RL filter.

1) Apply a sharp 5V square-wave signal (the TTL output of your signal generator) to your low-pass filter circuit and observe the resulting time response with an oscilloscope. Adjust the period of the square wave to be long enough that the circuit voltage appears to decay completely, but not much longer. Adjust the vertical scale and the horizontal sweep time so that a single voltage-decay curve approximately fills the display region. The response time of your circuit (τ) can be quantified as the time that it takes for the circuit voltage to complete 63% of its decay (i.e. fall to 37% of its original value). Visually measure τ and record it in your lab notebook.

2) Demonstrate that the product of the response time and the 3db cutoff frequency (i.e. $2\pi f_{3\text{db}}\tau$) is approximately equal to 1. Think carefully about this result and make some highly-intelligent comments about the relationship between time and frequency response curves?

C: Comparison to optical phenomena.

In the diffraction lab that you completed previously, you observed that *smaller* slit sizes or spacings in a source mask gave rise to *larger* fringe spacings in a diffraction pattern. This inverse relationship is highly analogous to the relationship between time signals and their frequency responses, and also has its origins in the Fourier transform. In fact, a diffraction pattern is essentially the frequency response of a spatial pattern rather than of a temporal pattern. It might help to think of a diffraction grating as a square-wave in space rather than in time. In your lab notebook, use your own words to describe the analogy between temporal and spatial frequency response functions.