

# PHYSICS 145 Homework Assignment on Complex Numbers

The purpose here is to help you to review your knowledge of complex numbers and complex arithmetic. You may use a scientific calculator, doing as much work by hand as possible. If you do your work on other sheets of paper, be sure to staple them to this assignment.

## **Complex addition and subtraction**

Examples:  $(5 + 3i) + (-2 + 7i) = 3 + 10i$        $(-6 + 14i) - (2 + 7i) = -8 + 7i$

1)  $(-1 + 2.7i) + (13.9 - 0.5i)$

2)  $(\sqrt{3} + 8i) + (-2\sqrt{3} - 7i)$

## **Complex multiplication**

Examples:  $(2 + 4i)(3 - 6i) = 30$        $(-1.5 + i)(7 - 3i) = -7.5 + 11.5i$

3)  $i(-1 + i)$

4)  $(3 + 4i)(3 - 4i) / 25$

5)  $1 + i + i^2 + i^3 + i^4 + i^5 + i^6 + i^7$

## **Complex division**

Examples:  $\frac{(2 + 4i)}{(3 - 6i)} = -0.4 + 0.533i$        $\frac{(-1.5 + i)}{(7 - 3i)} = -0.233 + 0.043i$

6)  $1/i$

7)  $2/(1 - i)$

8)  $(5.2 - 4.8i)/(0.6 + 0.8i)$

## **Absolute value (or modulus) and argument**

Examples:  $|3 + 4i| = 5$        $|1 - 7i| = 5\sqrt{2}$        $|\cos(\theta) + i \sin(\theta)| = 1$

$$\arg(-1 + i) = \operatorname{atan}(-1) + \pi = \frac{3\pi}{4} = 135^\circ \quad \arg(-3 - 2i) = \operatorname{atan}\left(\frac{2}{3}\right) - \pi = -146.3^\circ$$

9)  $|i|$

10)  $|4 + 3i|$

11)  $\arg(i)$

12)  $\arg(1.05 - 7.99i)$

## **Polar and Standard Forms**

Examples:  $-3 + 3i = 3\sqrt{2} e^{i(3\pi/4)}$        $-3 - 2i = 3.61 e^{-2.55i}$

$$7e^{i(3\pi/2)} = -7i \quad 4.5e^{-1.2i} = 1.63 - 4.19i \quad Ae^{i\theta} = A\cos(\theta) + iA\sin(\theta)$$

Convert to polar form.

13)  $-i$

14)  $5.71 - \sqrt{3}i$

Convert to standard form

15)  $2e^{i\pi/6}$

16)  $8.9e^{-100i}$

## **Complex functions**

Define  $u = 1 + i/\omega$  and  $v = 2e^{i\omega}$ , where  $\omega$  is a real number. You may use symbolic mathematical software for these problems if you wish. Otherwise, rough hand-drawn plots are fine.

17) Plot  $|u|$  as a function of  $\omega$  over the range from -3 to 3.

18) Plot  $\arg[u]$  (in radians) as a function of  $\omega$  over the range from -3 to 3.

19) Plot  $\operatorname{Re}[v]$  as a function of  $\omega$  over the range from  $-\pi$  to  $\pi$ .

20) Plot  $\operatorname{Im}[v]$  as a function of  $\omega$  over the range from  $-\pi$  to  $\pi$ .