In class I said that the Klein-Gordon equation for a spinless particle of mass $m$ and charge $-e$ in the presence of an infinitely massive nuclear charge $+Ze$ gives the energies

$$E_{n\ell} = \frac{mc^2}{\left\{ 1 + \frac{(Z\alpha)^2}{n - \ell - \frac{1}{2} + \sqrt{(\ell + \frac{1}{2})^2 - (Z\alpha)^2}} \right\}^{1/2}}$$

where $\alpha$ is the fine-structure constant, and $n$ and $\ell$ have their usual meanings.

1. Expand this equation as a power series in $(Z\alpha)^2$.

2. Show explicitly that the first three members of the power series are the rest energy of the particle, the Bohr energy of the particle (Equation 4.70 on page 137 in your text), and the first-order relativistic correction (Equation 6.56 on page 238 in your text).

3. How should we treat the spin-orbit interaction in the Klein-Gordon equation?