Review notes for Physics 220 Final Exam

Carefully look through the review notes for each of the five midterm exams. The final exam is comprehensive, though roughly 25% of the questions will address new concepts introduced after midterm exam 5. The review notes below pertain only to the new material.

Review notes on recent concepts (electromagnetic waves)

1) Displacement current and the Ampere-Maxwell law.
   a. Maxwell added the missing term in 1861: \( \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \). Just as a changing magnetic flux creates E-field loops, a changing electric flux creates B-field loops.
   b. Maxwell introduced another conceptual innovation, calling his extra term a “displacement current”: \( I_d \equiv \varepsilon_0 \frac{d\Phi_E}{dt} \Rightarrow \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) \). A changing E-field is a “current-like” quantity that provides continuity across the gap of a capacitor.
   c. Calculate \( \mathbf{B}(r) \) inside a parallel plate capacitor during charging or discharging.
   d. Write each of Maxwell’s equations and the Lorentz force law in equation form. Also state each of them in plain English without reference to symbols or acronyms including the directional information.

2) Waves
   a. A classical wave is a function \( f(x,t) \) that obeys the classical wave equation \( \frac{d^2 f}{dx^2} = \frac{1}{v^2} \frac{d^2 f}{dt^2} \).
   b. The parameters of a traveling plane wave: \( A \sin(\mathbf{k} \cdot \mathbf{x} - \omega t) \)
      i. Amplitude (A) and phase (\( \phi = \mathbf{k} \cdot \mathbf{x} - \omega t \))
      ii. Spatial: wavelength (\( \lambda \)), wavevector (\( \mathbf{k} = 2\pi/\lambda \), rad/m), wavenumber (\( 1/\lambda \), cycles/m).
      iii. Time: period (\( T \)), angular frequency (\( \omega = 2\pi/T \), rad/s), frequency (\( \nu = 1/T \), cycles/s).
      iv. Velocity = \( \omega/k \). The sign determines the direction of the wave.
   c. The function \( \omega(k) \) is called a dispersion relation – simple if velocity is a constant.
   d. A vector amplitude (displacement, field, etc.) can be parallel (longitudinal) or perpendicular (transverse) to \( \mathbf{k} \).
   e. A transverse wave has polarization (linear or circular).
   f. Waves can be linearly superimposed to obtain a new wave. Two otherwise identical waves with opposite velocities superimpose to form a standing wave. The distance between nodes in a standing wave is \( \lambda/2 \).

3) Electromagnetic waves
   a. Light is an electromagnetic wave (i.e. a propagating combination of E and B fields).
   b. Light’s dispersion relation is \( c = \omega/k = \lambda \nu \), where \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8 \) m/s.
   c. An observer always measures light to have the same speed in a vacuum regardless of their reference frame of motion. This bizarre fact, deduced from the experiments of Michaelson and Morley, led us to abandon the idea of a physical medium (the ether) for light waves and inspired Einstein’s theory of special relativity.
   d. The ratio of field strengths in a light wave is \( \frac{E}{B} = c \).
   e. Yet the E and B fields carry equal energy densities \( u_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2} \varepsilon_0 E^2 = u_E \).
   f. Total energy density: \( u = u_E + u_B = \frac{1}{2\mu_0} B^2 + \frac{1}{2} \varepsilon_0 E^2 = 2u_E = \varepsilon_0 E^2 = 2u_B = \frac{1}{\mu_0} B^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}} E \cdot B \)
g. The energy flow vector of an E&M wave (also called the Poynting vector) has units of intensity ($J/m^2/s$), and points along the propagation direction. It is defined as
\[ \mathbf{S} = \mathbf{E} \times \mathbf{H} = \mathbf{E} \times \frac{\mathbf{B}}{\mu_0}, \]
implying $\mathbf{E}$ and $\mathbf{B}$ are perpendicular. This energy-current density $\mathbf{S} = \mathbf{u}c$ is analogous to charge-current density $\mathbf{J} = \rho \mathbf{v}$ (density*velocity).

h. $\mathbf{E}$ and $\mathbf{B}$ can be defined in terms of either max or rms values. Energy densities and flow vectors can be defined in terms of either max or ave values. Here are a few relationships to consider.

i. \[ \bar{u} = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_{rms} B_{rms} = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} E_{max} B_{max} = \frac{1}{2} u_{max} \]

ii. \[ \bar{\mathbf{S}} = c \bar{u} = \frac{1}{\sqrt{\mu_0}} E_{rms} B_{rms} = \frac{1}{2} \sqrt{\mu_0} E_{max} B_{max} = \frac{1}{2} S_{max} = \frac{1}{2} cu_{max} \]

i. Light has momentum: $U = p c$ and pressure: $P = \frac{S}{c}$.

j. The radiation pressure (force/area) exerted on surface has two special cases: reflection $P = \frac{2S}{c}$ and absorption $P = \frac{S}{c}$.

4) Dipole Radiation

a. Dipole radiation intensity falls off as $1/r^2$, but also has an angular dependance $\sin^2(\theta)$, so that no energy flows along the axis of the antenna. Thus the $\mathbf{E}$ and $\mathbf{B}$ fields are most intense in the plane perpendicular to the dipole of the transmitter.

b. In the plane perpendicular to a electric dipole transmitter, the polarization of the $\mathbf{E}$-field will be parallel to the transmitter's electric dipole direction. Similarly, for a magnetic dipole transmitter, the polarization of the $\mathbf{B}$-field will be parallel to the transmitter's magnetic dipole direction.

c. For an electric dipole antennae, the ideal antennae lengths are $\lambda/2$ for center-fed antennae and $\lambda/4$ for end-fed antennae.

d. Optimal reception is obtained by aligning the antenna properly relative to the transmitter dipole. An electric dipole antenna should be aligned parallel to the $\mathbf{E}$-field polarization. A magnetic dipole antenna (i.e. a coil) should be aligned parallel to the $\mathbf{B}$-field polarization.