How many of these trajectories belongs to a negatively charged particle? A. 1 B. 2 C. 3 D. 4 or more E. zero
Velocity Selector

![Diagram of Velocity Selector]

\[ F_B = F_E \]
\[ qvB = qE \]
\[ v = \frac{E}{B} \]

Mass spectrometer

![Diagram of Mass Spectrometer]

Separately measure \( v \) and \( r \).

\[ \frac{q}{m} = \frac{v}{Br} \]
Cathode ray tube

\[ F_B = F_E \implies qvB = qE \]

\[ \implies v = \frac{E}{B} \]
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| Nov 9   | W    | 31:5-6 Lab 9 description | Motional emf --sliding bar/moving loop  
Eddy currents  
DC homopolar motor | 31:C-D 25                     |
iClicker Quiz

(1) I have completed at least 50% of the reading and study-guide assignments associated with the lecture, as indicated on the course schedule.

a) True  b) False

Today: How B fields are created by currents. (Worst vector integral you will (not) have to do.)

First some review. Hall effect HW problem. Then A. Biot-Savart Law & applications B. straight line (segments), C. loops & partial loops. Also: (How to draw & understand drawings.) Figure out how & when to use the right hand rule to get direction of vector ds and r

Hint: This is a good time to see that the equations you want are in your CS.
To get this one right you must use the real drift direction of the electrons.

**Which direction do the electrons flow?**  pp.

A. Up  B. Down  C. Into the screen  D. Out of the screen.
Wire moves: Doesn’t matter if current carriers are positive or negative. Either way, the force direction is the same.

Potential Across wire: Current flowing through conducting object in the presence of $\perp B$ field. Positive & negative carriers would be forced in _____ direction.

For a metallic wire, $\Delta V = V_{\text{top}} - V_{\text{bottom}}$ should be (A) positive (B) negative (C) zero?
Case #1: Electric current flow in a wire.

Case #2: Fluid mixture of positive and negative ions flowing through a tube.

Which terminal is more positive?

(1) A (2) B (3) $\Delta V = 0$
Hall Effect: Quantitative

\[ I = J A = n_c q_c v_d t d \]

\[ \Rightarrow \quad v_d = \frac{I}{n_c q_c t d} \]

\[ F = q_c (v_d B) = q_c [E_{eff}] \]

\[ V_H = (v_d B)d = [E_{eff}]d \]

\[ V_H = q_c \left( \frac{I}{n_c q_c t d} \right) Bd = \frac{IB}{n_c q_c t} \]
The electric field diverges outward from a point charge.

\[ d\vec{E} = k_e \frac{dQ \hat{r}}{r^2} \]

The magnetic field circulates around an electric current (RHR).

\[ d\vec{B} = ? \]
Biot-Savart Law

\[ d\mathbf{B} = k_m \frac{(dQ \, \mathbf{v}) \times \mathbf{r}}{r^2} \]

\[ = k_m \frac{(Ids) \times \mathbf{r}}{r^2} \]

\( d\mathbf{B} \) is proportional to \( \sin(\theta) \).

\[ k_m = \frac{\mu_0}{4\pi} = 10^{-7} \frac{T \cdot m}{A} \]

\[ \mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{ds \times \mathbf{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{ds \times \mathbf{r}}{r^3} \]
Biot-Savart example: long straight wire

$z$ runs from $-\infty$ to $+\infty$ as $\theta$ runs from $-\pi/2$ to $+\pi/2$.

$\hat{z}$ points along current direction.

$\hat{\rho}$ points radially away from wire.

$\hat{\phi} = \hat{z} \times \hat{\rho}$ circulates around wire.

If $\hat{\rho} = \hat{x}$, then $\hat{\phi} = \hat{z} \times \hat{x} = \hat{y}$.

$$\begin{aligned}
    ds &= dz \hat{z} \\
    r &= \sqrt{z^2 + a^2} \\
    \hat{r} &= \frac{-z\hat{z} + a\hat{\rho}}{r} \\
    \cos(\theta) &= \frac{a}{r} = \frac{a}{\sqrt{z^2 + a^2}} \\
    &\implies dz = \frac{a \, d\theta}{\cos^2(\theta)} \\
    \frac{ds \times \hat{r}}{r^2} &= \frac{adz \hat{\phi}}{r^3} = a \left( \frac{\cos(\theta)}{a} \right)^3 \left( \frac{a \, d\theta}{\cos(\theta)} \right) \hat{\phi} = \frac{\cos(\theta) \, d\theta}{a} \hat{\phi}
\end{aligned}$$
Biot-Savart example: long straight wire

$z$ runs from $-\infty$ to $+\infty$ as $\theta$ runs from $-\pi/2$ to $+\pi/2$.

\[
\frac{ds \times \hat{r}}{r^2} = \frac{\cos(\theta) \, d\theta}{a} \hat{\phi}
\]

\[
d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{ds \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi a} \hat{\phi} \cos(\theta) \, d\theta
\]

\[
\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi a} \hat{\phi} \int_{-\pi/2}^{\pi/2} \cos(\theta) \, d\theta = \frac{\mu_0 I}{4\pi a} \left[ \sin(\theta) \right]_{-\pi/2}^{\pi/2} \hat{\phi} = \frac{\mu_0 I}{2\pi a} \hat{\phi}
\]
Biot-Savart example: finite wire segment

$z$ runs from $z_1$ to $z_2$ as $\theta$ runs from $\theta_1$ to $\theta_2$.

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I \hat{\phi}}{4\pi a} \int_{\theta_1}^{\theta_2} \cos(\theta) \, d\theta = \frac{\mu_0 I}{4\pi a} [\sin(\theta_2) - \sin(\theta_1)] \hat{\phi}$$
20-1. A loop of wire of length $L = 10.8$ cm is stretched into the shape of a square and carries a current of $I = [01]$ ________ A. Determine the magnitude of the magnetic field at the center of the loop due to the current-carrying wire. [10.0, 20.0 μT]

Do we know enough to do this problem?
A. Yes, B. No   C. but can we talk about it?
Biot-Savart example: square loop

B-field at $P$ due to current in wire segment #1 is directed
(1) into screen (2) out of screen
(3) left (4) right (5) up (6) down?

Wire segment #2?

All four segments contribute equally to the magnitude.

$$B_1 = \frac{\mu_0 I}{4\pi a} \left[ \sin(\theta_2) - \sin(\theta_1) \right] = \frac{\mu_0 I}{4\pi a} \left[ \sin(\pi / 4) - \sin(-\pi / 4) \right] = \frac{\mu_0 I}{2\sqrt{2}\pi a}$$

$$B = B_1 + B_2 + B_3 + B_4 = 4B_1 = \frac{\mu_0 I}{(\pi / \sqrt{2})a}$$
Biot-Savart example: circular loop

\[ \hat{z} \] points perp. \( \perp \) to the loop (left).

\[ \hat{r} \] points radially inward.

\[ \hat{\phi} \] circulates CCW with current.

\[
\begin{align*}
d\mathbf{B} &= \frac{\mu_0 I}{4\pi} \frac{\mathbf{ds} \times \hat{\mathbf{r}}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{\mathbf{ds} \hat{\phi} \times \hat{\mathbf{r}}}{a^2} = \frac{\mu_0 I}{4\pi} \frac{\mathbf{ds}}{a^2} \hat{\mathbf{z}} = \frac{\mu_0 I}{4\pi a} d\phi \hat{\mathbf{z}} \\
\mathbf{B} &= \int d\mathbf{B} = \frac{\mu_0 I}{4\pi a} \int_0^{2\pi} d\phi = \frac{\mu_0 I}{2a} \hat{\mathbf{z}}
\end{align*}
\]
20-2. A conductor consisting of a circular loop of radius \( R = \) [02] \( \text{m} \) and two straight, long sections, carries a current of \( I = 7.00 \text{ A} \). In the figure, the loop is viewed from the \( +z \) direction. Determine the \( z \) component of the resulting magnetic field at the center of the loop. \([-1.00, -3.00 \mu \text{T}]\)

**Do both the straight wire and the loop contribute B field?**
A. Yes  B. No  C. Can we talk about it?

**Which eqn should we use?**
A. \( B = \mu I/(2\pi a) \)
B. \( B = \mu I/(2a) \)  C. Both  D. Neither.  E. Something else
20-3. Consider the current carrying loop shown below, which is formed of radial lines and segments of circles whose centers are at point $P$ and whose radii are $a = 20.0$ cm and $b = 50.0$ cm. The current in the loop is $I = \phantom{0}03\phantom{0000}$ mA. Assuming that the loop is viewed from the $+z$ direction, determine the $z$ component of the resulting magnetic field at point $P$. $[0.90, \ 1.30 \text{ nT}]$

Thought question: Do both the straight wire and the loop contribute B field?
A. Yes  B. No  C. Can we talk about it?
Biot-Savart example: partial loop

B-field at $P$ due to current in wire segment 1 is directed.
(A) into screen  (B) out of screen  (C) some other direction?

Let $\theta = 45^\circ$

\[
B = 0 + \left( \frac{\theta}{2\pi} \right) \frac{\mu_0 I}{2a} + 0 = \left( \frac{\pi/4}{2\pi} \right) \frac{\mu_0 I}{2a} = \frac{\mu_0 I}{16a}
\]
Biot-Savart example: on axis of loop

\[ ds = a \, d\phi \, \hat{\phi} \quad r = \sqrt{z^2 + a^2} \quad \vec{r} = z \hat{z} - a \hat{\rho} \]

\[ \hat{\phi} \times \vec{r} = z (\hat{\phi} \times \hat{z}) - a (\hat{\phi} \times \hat{\rho}) = z \hat{\rho} + a \hat{z} \]

\[ dB_\rho \text{ integrates to zero} \]

\[ dB_z = \frac{\mu_0 I}{4\pi} \frac{(ds \times \vec{r})_z}{r^3} = \frac{\mu_0 I}{4\pi} \frac{(a \, d\phi \, \hat{\phi} \times \vec{r})_z}{r^3} = \frac{\mu_0 I}{4\pi} \frac{a^2 \, d\phi}{r^3} \]

\[ B_z = \int dB_z = \frac{\mu_0 I}{4\pi} \frac{a^2}{r^3} \int_0^{2\pi} d\phi = \frac{\mu_0 I}{2} \frac{a^2}{r^3} = \frac{\mu_0 I}{2} \frac{a^2}{(z^2 + a^2)^{3/2}} \]
Biot-Savart example: loop axis \((z \gg a)\)

\[
B_z = \frac{\mu_0 I}{2} \frac{a^2}{(z^2 + a^2)^{3/2}} \quad \text{for} \quad z \gg a \rightarrow \frac{\mu_0 I}{2} \frac{a^2}{z^3}
\]

Use the point-dipole formula:

\[
B = k_m \frac{3(\mu \cdot \hat{r})\hat{r} - \mu}{r^3} = k_m \frac{3(\mu \hat{z} \cdot \hat{z})\hat{z} - \mu \hat{z}}{z^3} = \frac{2k_m \mu}{z^3} \hat{Z} = 2 \frac{\mu_0}{4\pi} \frac{NIA}{z^3} \hat{Z} = \frac{\mu_0 I}{2} \frac{a^2}{z^3} \hat{Z}
\]
Reading Quiz: when wires carry currents in the same direction there are forces:

A. The draw the two wires together.  B push the wires apart.  C. neither.
Parallel current-carrying wires

The first wire produces a field that is experienced by the second wire.

$$B_{21} = \frac{\mu_0 I}{2\pi a}$$

$$F_{21} = I_2 \ell B_{21} = \frac{\mu_0 I_1 I_2 \ell}{2\pi a}$$

To achieve a mutually attractive force, the two currents should be (A) parallel (B) antiparallel?
**Note on Monday, 17th’s reading: 30:7 is part of 29.1 in 8th edition**

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(1) I have completed at least 50% of the reading and study-guide assignments associated with the lecture, as indicated on the course schedule.
   a) True
   b) False

Today: How B fields are created by currents in very symmetric cases. Ampere’s Law (Another law that uses an integral to get a field.) First some review. (Loops.) and attraction of wires. Then A. Ampere’s Law & applications B. Long straight line. C. solenoids & toroids. D. loops? Also: (How to draw & understand drawings.) Hint: This is a good time to see that the equations you want are in your CS.
\[ \Delta V = -\oint E \cdot ds = 0 \quad \text{Kirchhoff's voltage rule} \]

Gauss’s law of electrostatics: \( E \)-field lines start on \( + \) charges and end on negative charges. \text{No loops!}
Oersted’s $B$ field loops around a current-carrying wire.

Static $B$ fields are not “conservative”. 

$$\oint B \cdot ds \neq 0$$
Ampere’s Law

Magnetic circulation $\mathcal{C} \equiv \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$

Andre-Marie Ampere

The magnetic circulation around a closed loop is proportional to the electric current flowing through the loop, where the positive directions of electric current ($I$) and magnetic circulation ($\mathcal{C}$) are related by the RHR.
Magnetic circulation quiz

Highest magnetic circulation (consider the sign)? pp.

Largest magnetic circulation (magnitude only)?

Which one has zero magnetic circulation?
Magnetic circulation quiz

Highest magnetic circulation? (Consider the sign.)

Largest magnetic circulation? (magnitude only)
Which loop has the largest circulation?
21-1. Two square current-carrying loops and two closed integration paths, one dashed and one solid, are arranged as shown. If the positive current direction is chosen to be clockwise, the current in the loop on the left is +10.0 A. Defining $\xi = \int \mathbf{B} \cdot d\mathbf{s}$ for a given path, we find that the ratio $\xi_{\text{dashed}}/\xi_{\text{solid}} = [01]$ __________. Determine the current (magnitude and sign) in the right-hand loop. Hint: Draw a top-view diagram of the figure, which should make the looped paths and current directions more apparent. $[-90.0, 90.0 \text{ A}]$

Is the contribution to the dotted loop from the 10A current **positive (A)** or negative **B** or **Zero (C)**?

Is the contribution to the **solid loop** from the 10A current **positive (A)** or negative **B** or **Zero (C)**? pp
Inverting Ampere’s law to determine $B$ requires a high symmetry situation.

**Ampere’s Law example: long straight wire**

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint d\mathbf{s} = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Inverting Ampere’s law to determine $B$ requires a high symmetry situation.
21-2. In the cross-sectional view of a coaxial cable below, the center conductor is surrounded by a rubber layer, which is surrounded by an outer conductor, which is surrounded by another rubber layer. In a particular application, the current in the inner conductor is \( I_{\text{inner}} = [02] \text{ mA} \), directed out of the page, while the current in the outer conductor is \( I_{\text{outer}} = [03] \text{ mA} \), directed into the page. Determine magnitude and sign of the vertical (up = +) component of (a) the magnetic field at point \( a \) and (b) the magnetic field at point \( b \). [(a) –40.0, 40.0 \( \mu \text{T} \) (b) –40.0, 40.0 \( \mu \text{T} \)]
Reading Quiz: For which of the following situations can Ampere’s law NOT be easily inverted to get the B field?

A. Long straight wire
B. Inside a long, straight, thick wire
C. A solenoid
D. A toroid
E. A loop of wire.
Ampere’s Law example: inside a thick wire

Inside this wire, the enclosed current increases linearly with $r$. 

\[
\oint B \cdot ds = B \oint ds = B(2\pi r) = \mu_0 I_{\text{enc}} \\
I_{\text{enc}} = JA_{\text{enc}} = I \frac{A_{\text{enc}}}{A} = I \left( \frac{r}{R} \right)^2 \\
B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} = \frac{\mu_0 I}{2\pi R^2} 
\]
Ampere’s Law example: inside a solenoid

\[ \oint \mathbf{B} \cdot d\mathbf{s} = Bl + 0 + (0)l + 0 = Bl = \mu_0 NI \]

\[ B = \mu_0 \frac{N}{\ell} I = \mu_0 nI \]
21-3. A superconducting solenoid with 2000 turns/m is meant to generate a magnetic field of \[0.4\] T. (a) Calculate the current required. (b) Determine the force per unit length exerted on the windings by this magnetic field. Note that while an individual current-carrying wire segment experiences no force due to the $B$-field that it creates, that wire segment does experience a force due to the collective field produced by all of the current-carrying coils around the solenoid. [(a) 3.00, 6.00 kA (b) 30.0, 90.0 kN/m]
Reading Quiz: For which of the following situations can Ampere’s law NOT be easily inverted to get the B field?

A. Long straight wire
B. Inside a long, straight, thick wire
C. A solenoid
D. A toroid
E. A loop of wire.

Now it counts
Ampere’s Law example: inside a toroid

\[ \oint \mathbf{B} \cdot d\mathbf{s} = B \oint d\mathbf{s} = B(2\pi r) = \mu_0 (NI) \]

\[ B = \frac{\mu_0 NI}{2\pi r} \]
Use Ampere’s law to determine $B$.

There’s not enough symmetry.
21-4. Complete this problem on a separate sheet of paper and submit it with your CID# prominently displayed.

Two long parallel conducting wires are shown below in cross section. Both conductors carry equal currents that are uniformly distributed over their respective cross sections. The conductor on the right always carries current into the page. Sketch the $y$ component of the magnetic field along the $x$ axis from $x = -5a$ to $x = +5a$ under the assumption that the conductor on the left carries current (a) in the same direction as the conductor on the right and (b) in the opposite direction as the conductor on the right.
\[ \frac{1}{\mu_0} \int \mathbf{B} \cdot ds = ? \]
Current density $\mathbf{J}$ flowing uniformly along $+z$ direction. What’s the magnetic circulation around this loop?

This tube or radius $r$ is coated with uniform positive surface charge density $\sigma$ and rotates around its axis with angular velocity $\omega$, so as to produce a uniform magnetic field $B$ inside the tube? Find the magnitude of $B$?