iClicker Quiz

(1) I have completed at least 50% of the reading and study-guide assignments associated with the lecture, as indicated on the course schedule.
A. True  B. False

Today there will be many, many new terms: $I, \mathbf{j}, \tau$ (tau), $v$ (drift), $\sigma$ (sigma), $\rho$ (rho), $R$, Power and several equations:
1. $I\& \mathbf{j}$  2. microscopic theory of conduction  3.
In static equilibrium, \( E = 0 \) inside a conducting material.

Now consider non-equilibrium situations where \( E \neq 0 \) and \( \Delta V \neq 0 \) in a conductor.

A nonzero \( E \)-field inside a conductor drives a nonzero current density \( J \), which requires a continuous source of flowing electric charge (e.g. a battery or power supply).

When the electric current in a conductor is constant in time, we do not say that the charges present are in equilibrium, but instead say that they are in a “steady state”.
Electric Current

• **Electric current** is the rate of flow of charge through some region of space.
• The SI unit of current is the **ampere** (A).
  – $1 \text{ A} = 1 \text{ C} / \text{s}$
• The symbol for electric current is $I$. 
Average Electric Current

- Assume charges are moving perpendicular to a surface of area $A$.
- If $\Delta Q$ is the amount of charge that passes through $A$ in time $\Delta t$, then the average current is

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$
Instantaneous Electric Current

- If the rate at which the charge flows varies with time, the instantaneous current, \( I \), is defined as the differential limit of average current as \( \Delta t \to 0 \).

\[
I \equiv \frac{dQ}{dt}
\]
$I = \int \mathbf{J} \cdot d\mathbf{A}$

$I =$ current or charge flux through a defined area
scalar quantity (coulombs/sec)

$\mathbf{J} =$ current density
vector quantity (coulombs/sec/m$^2$)

Analogous to $E$-field flux: $\Phi = \int \mathbf{E} \cdot d\mathbf{A}$
12-2. Suppose that the current through a conductor decreases exponentially with time according to the expression $I(t) = I_0 e^{-t/\tau}$, where $I_0$ is the initial current equal to 1.321 mA and $\tau$ is a constant equal to 0.02 s. Consider a piece of the conductor. (a) How much charge passes through this piece between $t = 0$ and $t = \tau$? (b) How much charge passes through this piece between $t = 0$ and $t = 4\tau$? (c) How much charge passes through this piece between $t = 0$ and $t = \infty$? [(a) 1.00, 4.00 mC (b) 1.00, 4.00 mC (c) 1.00, 4.00 mC]

This $\tau$ is not the same tau that you will meet in a minute. Do you know enough to start the problem?

What is the basic equation you will use?
## Electric-fields vs flow-related quantities

<table>
<thead>
<tr>
<th></th>
<th>carrier density</th>
<th>flux density (current density)</th>
<th>Flux (current)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho$</td>
<td>$\vec{J} = \rho \vec{v}$</td>
<td>$\vec{J} \cdot A$</td>
</tr>
<tr>
<td><strong>Fluid flow</strong></td>
<td>kg/m$^3$</td>
<td>kg/m$^2$/sec</td>
<td>kg/sec</td>
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<tr>
<td><strong>Electricity flow</strong></td>
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<td><strong>Energy flow</strong></td>
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<td><strong>Electric fields</strong></td>
<td>Fluxons/m$^3$</td>
<td>Electric field Fluxons/m$^2$/sec (N/C = Volts/m)</td>
<td>Fluxons/sec (Volt·m)</td>
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Fluxon (made it up by Campbell) = kg·m$^3$/C·sec → The flow analogy requires imagination.
Direction of Current

• The charged particles passing through the surface could be positive, negative or both.
• It is conventional to assign to the current the same direction as the flow of positive charges.
• In an ordinary conductor, the direction of current flow is opposite the direction of the flow of electrons.
• It is common to refer to any moving charge as a charge carrier.

Section 27.1
Rank the four regions in order of current magnitude. Which is largest? pp.

Which is smallest? Counts.
Now define the positive direction to be to the right. Rank the four regions in order of signed current. Which is largest?
• When a potential difference is applied across the conductor, an electric field is set up in the conductor which exerts an electric force on the electrons.
• The motion of the electrons is no longer random.
• The zigzag black lines represents the motion of a charge carrier in a conductor in the presence of an electric field.
  – The net drift speed is small.
• The sharp changes in direction are due to collisions.
• The net motion of electrons is opposite the direction of the electric field.
Microscopic theory of conduction

\[ \rho_c = n_c q_c \]  
\[ (\text{carriers/m}^3)(\text{charge/carrier}) = \text{charge/m}^3 \]

\[ J = \rho_c v_d = n_c q_c v_d \]  
\[ (\text{charge/m}^3)(\text{m/sec}) = \text{charge/sec/area} \]
Motion of Charge Carriers, con’t.

• In the presence of an electric field, in spite of all the collisions, the charge carriers slowly move along the conductor with a drift velocity, \( v_d \).

• The electric field exerts forces on the conduction electrons in the wire.
• These forces cause the electrons to move in the wire and create a current.
Motion of Charge Carriers, final

• The electrons are already in the wire.
• They respond to the electric field set up by the battery.
• The battery does not supply the electrons, it only establishes the electric field.
Drift Velocity, Example

• Assume a copper wire, with one free electron per atom contributed to the current.
• The drift velocity for a 12-gauge copper wire carrying a current of 10.0 A is

  \[ 2.23 \times 10^{-4} \text{ m/s} \]

  – This is a typical order of magnitude for drift velocities.
You are about to meet that other tau.

Can you guess what it is called?
Microscopic theory of conduction

 Resistivity is determined entirely by microscopic parameters including the average thermal collision rate \((1/\tau)\).

1. \( \mathbf{v}_d^E = a \tau = \left( \frac{q_c}{m_c} E \right) \tau = \frac{q_c \tau}{m_c} E \)

2. \( J = (n_c q_c) \left( \frac{q_c \tau}{m} E \right) = \frac{n q_c^2 \tau}{m_c} E = \sigma E \)

3. \( \rho = \frac{1}{\sigma} = \frac{m_c}{n_c q_c^2 \tau} \)
12-1. A uniform metallic rod, with a cross-sectional area of 1.83 cm² and a length of 7.08 m, contains $6.24 \times 10^{28}$ conduction electrons per cubic meter of material, which have a mean collision time of [01] _________ femtoseconds. (a) Determine the resistivity of the rod. When the rod experiences a potential difference of 2.52 mV from end to end, determine (b) the drift velocity of the electrons and (c) the current density in the rod.

[(a) $1.50 \times 10^{-8}$, $3.00 \times 10^{-8}$ Ω·m (b) $1.00 \times 10^{-6}$, $2.00 \times 10^{-6}$ m/s (c) 10000, 20000 ± 100 A/m²]

Which eqn works for part a?

A

$v_d = a\tau = \left(\frac{q_c}{m_c} E\right)\tau = \frac{q_c\tau}{m_c} E$

B

$J = \left(n_c q_c\right)\left(\frac{q_c\tau}{m} E\right) = \frac{n q_c^2 \tau}{m_c} E = \sigma E$

C

$\rho = \frac{1}{\sigma} = \frac{m_c}{n_c q_c^2 \tau}$
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(c) 10000. 20000 ± 100 A/m²]

Which eqn works for part b?

A \[ v_d = a \tau = \left( \frac{q_c}{m_c} E \right) \tau = \frac{q_c \tau}{m_c} E \]

B \[ J = \left( n_c q_c \right) \left( \frac{q_c \tau}{m} E \right) = \frac{n q_c^2 \tau}{m_c} E = \sigma E \]

C \[ \rho = \frac{1}{\sigma} = \frac{m_c}{n_c q_c^2 \tau} \]
iClicker Quiz

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Today:
A. ohm’s law. B. Effect of temperature of resistivity
C. ✤ POWER ✤ ✤ ✤ ✤ &
D. How to add up resistors in series and parallel: Friday
Ohm’s Law

Ohm’s law states that for many materials, the ratio of the current density to the electric field is a constant $\sigma$ that is independent of the electric field producing the current.

- Most metals obey Ohm’s law
- Mathematically, $J = \sigma E$
- Materials that obey Ohm’s law are said to be *ohmic*
- Not all materials follow Ohm’s law
  - Materials that do not obey Ohm’s law are said to be *nonohmic*.

Ohm’s law is not a fundamental law of nature.

Ohm’s law is an empirical relationship valid only for certain materials.
Ohm’s Law: linear response

\[ E = J \rho \quad \text{Local} \]

\[ I = \int J \cdot A = JA = \frac{E}{A} = \frac{(V / L)}{\rho} A \]

\[ V = I \left( \rho \frac{L}{A} \right) = IR \quad \text{Global} \]

\[ R = \rho \frac{L}{A} \]

Analogous to Poiseuille’s Law of fluid flow (hyperphysics).
Resistance

• In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor.

• The constant of proportionality is called the resistance of the conductor.

\[ R \equiv \frac{\Delta V}{I} \]

• SI units of resistance are ohms (Ω).
  \[ 1 \, \Omega = 1 \, \text{V} / \text{A} \]

• Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor.
We lack an important constant. What is it?
<table>
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$^a$ All values at 20°C. All elements in this table are assumed to be free of impurities.

$^b$ See Section 27.4.

$^c$ A nickel–chromium alloy commonly used in heating elements.

$^d$ The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.
This time:

A: Ohmic

non-Ohmic

B. Effect of temperature of resistivity

C. POWER &

D. How to add up resistors in series and parallel: Friday
Ohmic

\[ \text{Slope} = \frac{1}{R} \]

non-Ohmic
Electric Current Flow Quantities

**The resistor**

**MICROSCOPIC**
- \( q_c \) = carrier charge
- \( n_c \) = carrier density
- \( m_c \) = carrier mass

\( \tau \) = mean time between collisions

**LOCAL**
- \( \rho_c \) = charge density
- \( \sigma \) = conductivity
- \( \rho \) = resistivity

**GLOBAL**
- \( L \) = length
- \( A \) = area
- \( R \) = resistance

**The flow**

**MICROSCOPIC**
- \( v_d \) = drift velocity
- \( a \) = acceleration

**LOCAL**
- \( J \) = current density
- \( E \) = electric field

**GLOBAL**
- \( V \) = potential difference
- \( I \) = current
Assume that the charge \((q_c)\), mass \((m)\), density of carriers \((n_c)\) and mean collision time \((\tau)\) in a wire are known.

Which of the following quantities can be determined?

1) Resistivity \((\rho)\)
2) Resistance \((R)\)
3) Current \((I)\)
4) Current density \((J)\)
5) Drift velocity \((v_d)\)
6) Electric field \((E)\)

If we are also given the shape of the wire \((L\) and \(A)\), which of these quantities can be determined?

If we are further given the potential difference \((V)\) across the wire, which of these quantities can be determined?

\[
\rho = \frac{1}{\sigma} = \frac{m_c}{n_c q_c^2 \tau}
\]
\[
v_d = \frac{q_c \tau}{m_c} E
\]
\[
J = \rho_c v_d = \sigma E
\]
\[
E = \frac{V}{L} = \rho J
\]
\[
I = \int \mathbf{J} \cdot d\mathbf{A} = JA
\]
\[
V = IR
\]
\[
R = \frac{\rho L}{A}
\]
\[ R = \frac{V}{I} = \frac{\rho L}{A} \]

Quiz: If we double the current through a copper wire of length \( L \) and area \( A \), the resistance will be:  pp.
(1) quadrupled  (2) doubled  (3) unchanged  (4) halved  (5) quartered

Quiz: If we double the length and the diameter of copper wire, its resistance will be:
(1) quadrupled  (2) doubled  (3) unchanged  (4) halved  (5) quartered.

Quiz: If we stretch a copper wire to twice its original length at constant volume, its resistance will be:
(1) quadrupled  (2) doubled  (3) unchanged  (4) halved  (5) quartered.
Work to move charge $q$ through a potential difference $V$:

$$W = qV$$

If the average kinetic energy of the carriers doesn’t increase, then the energy expended has been converted to heat via atomic collisions. Let power ($P$) be the rate at which energy is dissipated as heat.

$$P = \frac{dW}{dt} = V \frac{dq}{dt} = VI =$$

$$(IR)I = I^2R = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

$$P = I^2R = \frac{V^2}{R}$$
13-3. A toaster is rated at \[02\] \[\text{W}\] when connected to a 120-V source. (a) What current does the toaster carry? (b) What is its resistance? [(a) 4.00, 7.00 A (b) 10.0, 30.0 \[\Omega\]]

What equation works best for part b? pp. or talk

A. \[P = \frac{dW}{dt} = V \frac{dq}{dt} = VI = (IR)I = I^2R = V \left(\frac{V}{R}\right) = \frac{V^2}{R}\]

B, C \[P = I^2R = \frac{V^2}{R}\]
An electric car is designed to run off a 12.0-V battery with a total energy storage of \[03 \text{ J.}\] (a) If the electric motor draws 8.00 kW, what is the current delivered to the motor? (b) If the electric motor draws 8.00 kW as the car moves at a steady speed of 20.0 m/s, how far will the car travel before it is “out of juice”? [(a) 500, 900 A (b) 30.0, 60.0 km]

What equation works best for part a? pp. or talk

A. 

\[ P = \frac{dW}{dt} = V \frac{dq}{dt} = VI = \]

\[ (IR)I = I^2R = V \left( \frac{V}{R} \right) = \frac{V^2}{R} \]

B, C

\[ P = I^2R = \frac{V^2}{R} \]
\[ R = \frac{V}{I} = \frac{\rho L}{A} \quad \text{and} \quad P = I^2 R = \frac{V^2}{R} \]

Quiz: If we double the resistance of an Ohmic wire of length \( L \) and area \( A \), the power dissipated in the wire will be:
(1) quadrupled  (2) doubled  (3) unchanged  (4) halved  (5) quartered.
(Not enough information! We could fix either \( V \) or \( I \)?)

Quiz: If we double the length of an Ohmic wire of area \( A \) and resistivity \( \rho \), while holding the potential difference \( V \) across the wire constant, the power dissipated in the wire will be:
(1) quadrupled  (2) doubled  (3) unchanged  (4) halved  (5) quartered.
Non-Ohmic behavior

Diode

Varistor

Cell Membrane

http://www.wikipedia.com
http://www.onsemi.com
3-1. When the voltage across a certain conducting filament is doubled, the current flowing through it is observed to increase by a factor greater than two. What type of material could the conductor be made of? 

Hint: Consider the effects of heating.

1. copper
2. quartz
3. lead
4. silicon

Have we talked yet about the principles to do this problem?
Resistance and Temperature

• Over a limited temperature range, the resistivity of a conductor varies approximately linearly with the temperature.
  \[ \rho = \rho_0 [1 + \alpha (T - T_o)] \]

  \(- \rho_o \) is the resistivity at some reference temperature \( T_o \)
  
  • \( T_o \) is usually taken to be 20\(^\circ\) C
  • \( \alpha \) is the temperature coefficient of resistivity
    
    \(- \text{SI units of } \alpha \text{ are } \text{ }^\circ\text{C}^{-1} \)

• The temperature coefficient of resistivity can be expressed as
  \[ \alpha = \frac{1}{\rho_o} \frac{\Delta \rho}{\Delta T} \]

Section 27.4
Temperature Variation of Resistance

• Since the resistance of a conductor with uniform cross sectional area is proportional to the resistivity, you can find the effect of temperature on resistance.

\[ R = R_0[1 + \alpha(T - T_0)] \]

• Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material.
The resistivity of a metal increases as it gets warmer.

The resistivity of an insulator or semiconductors decreases as it gets warmer.

Practical definition of metal/insulator
slope of $\rho$ vs. $T > 0 \rightarrow$ metal
slope of $\rho$ vs. $T < 0 \rightarrow$ insulator
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3-1. When the voltage across a certain conducting filament is doubled, the current flowing through it is observed to increase by a factor greater than two. What type of material could the conductor be made of? Hint: Consider the effects of heating.

1. copper
2. quartz
3. lead
4. silicon

What kind of material is it?

a. Dielectric b. semiconductor c. metal
The resistance of a platinum wire is to be calibrated for low-temperature measurements.

A platinum wire with a resistance of \( \Omega \) at 20\(^\circ\)C is immersed in liquid nitrogen at 77 K (−196\(^\circ\)C). If the temperature response of the platinum wire is linear, what is the expected resistance of the platinum wire in the liquid nitrogen?

\( (\alpha_{\text{platinum}} = 3.92 \times 10^{-3}/\text{°C}) \) [0.100, 0.400 \( \Omega \)]
Superconductors ($\rho = 0$)
Apply 100 V to a 1 meter length of 12-gauge copper wire \((r = 1 \text{ mm})\).

**Cu** : \(\rho = 1.7 \times 10^{-8} \Omega \cdot \text{m} \quad A = \pi (0.001)^2 = 3.14 \times 10^{-6} \text{ m}^2\)

\[
R = \rho \frac{L}{A} = \frac{(1.7 \times 10^{-8})(1.0)}{(3.14 \times 10^{-6})} = 5.4 \times 10^{-3} \Omega
\]

\[
I = \frac{V}{R} = \frac{(100)}{(5.4 \times 10^{-3})} = 18.5 \times 10^3 \text{ A}
\]

\[
P = I^2R = (18.5 \times 10^3)^2 (5.4 \times 10^{-3}) = 1.85 \times 10^6 \text{ W}
\]

\[
J = \frac{I}{A} = \frac{(18.5 \times 10^3)}{(3.14 \times 10^{-6})} = 5.9 \times 10^9 \text{ A/m}^2
\]

\[
E = \rho J = (1.7 \times 10^{-8})(5.9 \times 10^9) = 100 \text{ V/m}
\]

\[
v_d = \frac{J}{\rho_c n_c q_c} = \frac{(5.9 \times 10^9)}{(8.5 \times 10^{28})(1.602 \times 10^{-19})} = 0.433 \text{ m/s}
\]

\[
\tau = \frac{m v_d}{q_c E} = \frac{(9.11 \times 10^{-31})(0.433)}{(1.602 \times 10^{-19})(100)} = 24.6 \times 10^{-15} \text{ s}
\]
Apply 100 V to a 1 meter length of 12-gauge copper wire ($r = 1$ mm).

$$m_{\text{atomic}} = 63.5 \text{ g/mol} \quad \rho_m = 8.96 \text{ g/cm}^3$$

$$c_m = 3.85 \text{ J/g/K} \quad H_{\text{melt}} = 13.14 \text{ kJ/mol} \quad H_{\text{vap}} = 300.5 \text{ kJ/mol}$$

$$T_{\text{melt}} = 1356 \quad T_{\text{boil}} = 2840 \text{ K}$$

$$m = \rho_m A L = (8.96 \text{ g/cm}^3)(3.14 \times 10^{-6} \text{ m}^3)(1 \text{ m}) = 28.13 \text{ g}$$

$$n_{\text{mol}} = m / m_{\text{molar}} = \frac{(28.13 \text{ g})}{(63.5 \text{ g/mol})} = 0.443 \text{ mol}$$

$$\Delta E_{w_1} = c_m m \Delta T_1 = (3.85 \text{ J/g/K})(28.13 \text{ g})(1356 - 300 \text{ K}) = 11.4 \text{ kJ}$$

$$\Delta E_{\text{melt}} = n_{\text{mol}} H_{\text{melt}} = (0.443 \text{ mol})(13.14 \text{ kJ/mol}) = 5.82 \text{ kJ}$$

$$\Delta E_{w_2} = c_m m \Delta T_2 = (3.85 \text{ J/g/K})(28.13 \text{ g})(2840 - 1356 \text{ K}) = 161 \text{ kJ}$$

$$\Delta E_{\text{vap}} = n_{\text{mol}} H_{\text{vap}} = (0.443 \text{ mol})(300.5 \text{ kJ/mol}) = 133 \text{ kJ}$$

$$\Delta t_{w_1} = \frac{P}{\Delta E_{w_1}} = 6.1 \text{ ms} \quad \Delta t_{\text{melt}} = 3.2 \text{ ms} \quad \Delta t_{w_2} = 87 \text{ ms} \quad \Delta t_{\text{vap}} = 72 \text{ ms}$$
So what do you call a length of 12-gauge copper wire connected across the terminals of a good 120 V power source?

A fuse!
Cost of power used for home lighting
(50 bulbs, 25 Watts each, 50% use, $0.10 per kW·hr)

\[ P = (50 \text{ bulbs}) \left( \frac{25 \text{ W}}{\text{ bulb}} \right) = 2250 \text{ W} \]

\[ I = \frac{P}{V} = \frac{25 \text{ W}}{120 \text{ V}} = 0.21 \text{ Amp/ bulb} \]

\[ \text{cost} = (50\%) (2.25 \text{ kW}) \left( \frac{$0.10}{\text{kW} \cdot \text{hr}} \right) \left( \frac{30 \cdot 24 \text{ hr}}{\text{month}} \right) = $81/ \text{ month} \]
High-Voltage Transmission Line

\( (V = 120\ \text{V} \ vs. \ V' = 120\ \text{kV}) \)

Power transmitted:

\[ P'_{\text{trans}} = V' I' = V I = P_{\text{trans}} \]

Resistive power loss:

\[ P_{\text{loss}} = I^2 R \]

\[
\frac{P'_{\text{loss}}}{P_{\text{loss}}} = \frac{I'^2 R}{I^2 R} = \left( \frac{P'_{\text{trans}}/V'}{P_{\text{trans}}/V} \right)^2 = \left( \frac{V}{V'} \right)^2 = \left( \frac{1}{1000} \right)^2 = 10^{-6}
\]
High-Voltage Transmission Line (aluminum)
$L = 100 \text{ km}, \ r = 0.5 \text{ cm}, \ P_{\text{trans}} = 10 \text{ MW}, \ V = 120 \text{ kV}$

Cable cost $= (10^5 \text{ m})(\$30/\text{m}) = \$3 \times 10^6$

$$R = \rho \frac{L}{A} = \frac{(2.82 \times 10^{-8})(10^5)}{\pi(0.005)^2} = 36 \Omega$$

$$I = \frac{P_{\text{trans}}}{V} = \frac{(10 \times 10^6)}{(110 \times 10^3)} = 91 \text{ A}$$

$$P_{\text{loss}} = I^2 R = (91)^2 (36) = 300 \text{ kW} \rightarrow 30 \$/\text{hr}$$

$$V_{\text{loss}} = IR = (91)(36) = 3280 \text{ V} \quad \frac{P_{\text{loss}}}{P_{\text{trans}}} = \frac{(3 \times 10^5)}{(1 \times 10^7)} = 3\%$$

Increasing $A$ and cable cost by factor of 3 (adds $2 \times 10^6 \$) will decrease $R$ and $P_{\text{loss}}$ by factor of 3 (saves 20 \$/hr). The reduced losses pay for the more expensive cable after 11.4 years.
Effective resistance: two resistors in series

\[ R_{eq} = \frac{V}{I} = \frac{V_1 + V_2}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2 \]

\( R_1 \) and \( R_2 \) experience the same current but different voltages. Largest \( R \) has largest \( V \).

\( R_{eq} \) is larger than either \( R_1 \) or \( R_2 \).
Effective resistance: two resistors in parallel

\[ R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2} = \frac{V}{\frac{V}{R_1} + \frac{V}{R_2}} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \]

\( R_{eq} \) is smaller than smallest of \( R_1 \) and \( R_2 \).

\( R_1 \) and \( R_2 \) experience the same voltage but different currents. Smallest \( R \) has largest \( I \).
Reduction of a resistive network

\[
\begin{align*}
R & \quad R \\
R & \quad R \\
R & \quad R
\end{align*}
\quad \Rightarrow \quad
\begin{align*}
R & \quad R \\
R & \quad R \\
R & \quad R
\end{align*}
\quad \Downarrow
\begin{align*}
2R & \\
2R & \\
2R
\end{align*}
\quad \Leftarrow
\begin{align*}
R/2 \\
R/2
\end{align*}
A circuit diagram with nodes labeled as 'a' and 'b'. The circuit consists of resistors with values of 5.0 Ω, 3.0 Ω, 4.0 Ω, 10 Ω, and 2.0 Ω. There is a 28 V voltage source connected to the circuit.
Reduction of a resistive network

![Reduction of a resistive network diagram](image-url)
Apply 42 V between $a$ and $c$.

What is $I$ between $a$ and $c$?
$I = 3\, \text{A}$

What is $V_{bc}$?
$V_{bc} = 6\, \text{V}$

What is $I_2$?
$I_2 = 2\, \text{A}$
The switch is initially open. When the switch is closed, the current measured by the ammeter will:

A. increase  B. decrease  C. stay the same  D. fall to zero.
Compare the brightness of the four identical bulbs in this circuit.

D is in parallel with a zero-resistance wire. The current will take the zero-resistance path and bypass D altogether.

A and B are in series. So they will burn equally bright. Together, they see the full battery voltage.

C experiences the full battery voltage, or twice the voltage experienced by A or B. So C is four times as bright.
If $R_1$ is removed, $R_2$ will glow
(1) more brightly.
(2) less brightly.
(3) same brightness as before.

Household devices are wired to run in parallel!
Strings of 50 Christmas lights in series. Assume ~100 V source and 25 W power consumption.

What is the resistance of a single bulb?

A. 2Ω  B. 4Ω  C. 8Ω  D. 10 Ω
Resistance of an object with arbitrary shape

End-to-end:

\[ dR = d \left( \frac{\rho \text{ length}}{\text{area}} \right) = \rho \frac{dz}{A(z)} = \frac{\rho}{\pi(b^2 - a^2)} \]

\[ R = \int dR = \frac{\rho}{\pi(b^2 - a^2)} \int_0^L dz = \frac{\rho L}{\pi(b^2 - a^2)} \]

Inside-out:

\[ dR = d \left( \frac{\rho \text{ length}}{\text{area}} \right) = \rho \frac{dr}{A(r)} = \rho \frac{dr}{2\pi r L} \]

\[ R = \int dR = \frac{\rho}{2\pi L} \int_a^b dr = \frac{\rho}{2\pi L} \ln \left( \frac{b}{a} \right) \]
Resistance of an object with arbitrary shape

\[ R = \int dR = \int \rho \frac{dL}{A} = \rho \int \frac{dz}{A(z)} = \rho \int \frac{dz}{\pi r^2(z)} \]

\[ = \rho \int_{0}^{L} \frac{dz}{\pi [a + (b-a)z/L]^2} = \frac{\rho}{\pi} \left( \frac{L}{b-a} \right) \int_{z=0}^{L} \frac{du}{u^2} \]

\[ = \frac{\rho}{\pi} \left( \frac{L}{b-a} \right) \left( \frac{1}{a} - \frac{1}{b} \right) = \frac{\rho L}{\pi ab} \]
Batteries as circuit elements

- A battery or other constant-voltage device (e.g. power supply) is usually the energy source in a direct-current (DC) circuit.

- The **positive** terminal of the battery has higher potential than the **negative** terminal.

- The *electromotive force* (emf, ε) of a battery is the *open-circuit* voltage between its terminals when no current is flowing.

- Ideally, the battery has no internal resistance of its own.

- We *often* idealize the wires in a circuit to have zero resistance.
Real batteries

1. Chemical energy: an electrolyte solution allows negative ions to flow toward and react with the anode (−), while positive ions flow toward and react with the cathode (+).

2. Charge build-up prevents the reaction from proceeding unless an external circuit allows electrons accumulating at the anode to return to the cathode.

3. Dead when the reactants are used up. Rechargeable if the anode/cathode reactions are reversible.

4. Internal resistance tends to increase with age, use and multiple recharge cycles.
Internal resistance in non-ideal batteries

Terminal voltage:
\[ V \equiv V_{ab} = \varepsilon - Ir < \varepsilon \]

Current:
\[ I = \frac{\varepsilon}{R + r} = \frac{V}{R} < \frac{\varepsilon}{R} \]

Two AA batteries yield a combined \( \varepsilon = 3 \) V. You observe a terminal voltage of 2.7 V while delivering 300 mA of current to an ultra-bright flashlight.

What is the load resistance (\( R \))? (1) 1\( \Omega \) (2) 3\( \Omega \) (3) 9\( \Omega \) (4) 10\( \Omega \)

What is the internal resistance of a single AA battery? (1) 0.5\( \Omega \) (2) 1\( \Omega \) (3) 2\( \Omega \) (4) 3\( \Omega \)
Typical Alkaline (Zn/MnO2)
   1.5 to 1.6 V open circuit, 1.1 to 1.3 V closed circuit.
   163 W-hr/kg (590 J/g) – 400 kJ total for AA
   0.034 (122.4 C) to 15 A-hr (54 kC) of charge depending on size.
   85% Capacity after 4 years of non-use.

Lithium Ion
   3.2 V open circuit, 2.5 to 3.0 V closed circuit.
   230 W-hr/kg (828 J/g) – 460 kJ total for AA
   0.160 (576 C) to 1.4 Amp-hrs (5.04 kC) of charge
   95% capacity after 5 years of non-use

NiCd
   1.2 V open circuit
   50 W-hr/kg (180 J/g) – 140 kJ total for AA
   70% capacity after one month of non-use (500-5000 cycles)

Zn-Air bus battery:
   200 W-hr/kg (720 J/g)
   320 kW-hr (1.15 GJ) of energy
Multi-loop circuits

**Branch**: An independent current path experiences only one current at a given moment. It may be a simple wire or may also contain one or more circuit elements connected in series.

**Junction**: A point where three or more circuit branches meet.

**Loop**: A current path that begins and ends at the same circuit point, traversing one or more circuit branches, but without ever passing the same point twice.
Multi-loop circuits

The circuit above has: 3 branches, 2 junctions, 3 loops.

To solve for 3 unknown branch currents, we need 3 equations.

To get these equations, use all but one ($2 - 1 = 1$) junction, and as many independent loops as needed ($3 - 1 = 2$).
Kirchoff’s current rule: $\sum I_n = 0$

**Current rule:** The total current flowing into a junction is zero. Arrows define positive branch-current directions. A current later determined to flow opposite its arrow is “negative”.

\[ +I - I_1 - I_2 = 0 \]
\[ +I - I_1 + I_2 = 0 \]
\[ +I + I_1 + I_2 = 0 \]
Kirchoff’s Voltage Rule: \[ \sum \Delta V_n = 0 \]

Voltage rule: The voltage changes around a loop sum to zero. Arrows define positive branch-current directions.

\[ \Delta V = +\varepsilon \] for a battery crossing from – to + terminal.

Use \[ \Delta V = -IR \] when crossing a resistor in the positive direction.
Use \[ \Delta V = +IR \] when crossing a resistor in the negative direction.

Alpine loop elevation

\[ +\varepsilon - I_1R_1 = 0 \]
Single-loop circuit example

\[ I = \frac{15}{5000} = 3 \text{ mA} \]

\[ V_A - V_{ground} = +20 - I(2000) - 30 - I(1000) \]

\[ = -10 - I(3000) = -10 - (0.003)(3000) \]

\[ = -19 \text{ V} \]
Multiloop circuit example

Bottom loop:
\[ I_2 = -I_1 \]

Substitute \( I_2 \) in junction Eq:
\[ I_3 = -2I_1 \]
\[ + I_2 - I_1 - I_3 = 0 \]

Substitute \( I_2 \) and \( I_3 \) in top loop:
\[ I_1 = -5/3 \]

Solve for currents:
\[ I_1 = -5/3, \ I_2 = 5/3, \ I_3 = 10/3 \]
Multiloop circuit example

\[ +6I_1 - 10 - 4I_2 - 14 = 0 \]
\[ I_2 = 1.5I_1 - 6 \]
\[ -2I_3 + 10 - 6I_1 = 0 \]
\[ I_3 = -3I_1 + 5 \]
\[ +I_1 + I_2 - I_3 = 0 \]
\[ I_1 + (1.5I_1 - 6) - (-3I_1 + 5) \]
\[ = 5.5I_1 - 11 = 0 \]

\[ I_1 = 2 \quad I_2 = -3 \quad I_3 = -1 \]