I have completed at least 50% of the reading and study-guide assignments associated with the lecture, as indicated on the course schedule.

A. True  B. False

Today paying attention to minus signs and Δ (changes) is very important.

1. Work: Gravitational and Electrical Energy
2. Change in Potential energy and kinetic energy.
3. Potential vs. potential energy:
4. Uniform fields, point charges and dipoles.
   1. When can you get rid of the Δ?

5. Calendar: HW 7 due Thursday a.m.
   1. HW 8 due Saturday 8:45 a.m.
   2. Math Rev Make up ends Friday.
Force is a vector. Differential path length is a vector. **Work is a scalar quantity.**

Only the force component along the motion direction does work.

\[ dW = \mathbf{F} \cdot d\mathbf{s} \quad W = \int_{a}^{b} \mathbf{F} \cdot d\mathbf{s} \]

Pulling a cart through deep sand (motion parallel to force).
Work done by gravity

How much work is done at point A? A.+, B-, C 0

Work done by \( F_g \): \[ dW_g = F_g \cdot ds \]
Change in gravitational potential energy = (−)work done by gravity

\[ dU_g = -dW_g = -F_g \cdot ds \]

\[ \Delta U_g = -W_g = -\int F_g \cdot ds = -\int F_{gx} \, dx \]

\[ = -\int (-mg) \, dx = mg \Delta x \]
Work done by gravity

\[ \Delta U_g = -W_g = - \int F_g \cdot ds = - \int F_{gx} \, dx \]

\[ = -(-mg \sin(\theta)) \int dx = mg \Delta x \sin(\theta) \]
Gravitational potential

\[ \Delta U_g = - \int F_g \cdot ds = -m \int g \cdot ds = mg \Delta x \]

\[ \Delta V_g = \frac{\Delta U}{m} = - \int \frac{F_g}{m} \cdot ds = - \int g \cdot ds = g \Delta x \]

Gravitational potential is not the same as potential energy. It’s the energy/mass that a test mass would possess if present. No test mass is needed for a gravitational potential to exist.

\[ U_{Mm} = \frac{G M m}{r} \quad V_M = \frac{G M}{r} \]
Force

\[ F = \frac{dU}{dx} \]

Potential Energy

\[ U = -\int F \, dx \]

\[ F = mg \]

\[ U = mV \]

\[ \frac{g}{m} = g \]

\[ V = \frac{U}{m} \]

\[ g = \frac{dV}{dx} \]

\[ V = -\int g \, dx \]

Field

Potential
Electric potential

\[ \Delta U_e = -\int F_e \cdot ds = -q\int E \cdot ds = qE \Delta x \]

\[ \Delta V_e = \frac{\Delta U_e}{q} = -\int \frac{F_e}{q} \cdot ds = -\int E \cdot ds = E \Delta x \]

Electric potential is not the same as potential energy. It’s the energy/charge that a test charge would possess if present. No test charge is needed for an electric potential to exist.

\[ U_{Qq} = \frac{kQq}{r} \quad V_Q = \frac{kQ}{r} \]
Electric Potential, final

• The difference in potential is the meaningful quantity.
• We often take the value of the potential to be zero at some convenient point in the field.
• Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.
• The potential difference between two points exists solely because of a source charge and depends on the source charge distribution.
  – For a potential energy to exist, there must be a system of two or more charges.
  – The potential energy belongs to the system and changes only if a charge is moved relative to the rest of the system.

Section 25.1
**Force**

\[ F = -\frac{dU}{dx} \]

**Potential Energy**

\[ U = -\int F \, dx \]

\[ F = qE \]

\[ U = qV \]

\[ E = F / q \]

\[ V = U / q \]

**Field**

\[ E = -\frac{dV}{dx} \]

**Potential**

\[ V = -\int E \, dx \]
Two kinds of fields to study

• Parallel plates
• Point charges.
Vector fields point downhill in potential ($V$).
Forces point downhill in energy ($U$).
Static $E$-fields are conservative, which implies that the potential difference between two points is independent of the path traveled!

An electron (or proton) passing through a 1 Volt potential difference experiences a 1 electron-Volt (eV) change in potential energy.
A free charge passing through a potential difference will experience a change in kinetic energy opposite to its change in potential energy.

$$\Delta U + \Delta K = 0$$

$$\Delta U = q\Delta V$$

$$\frac{1}{2}m(v_f^2 - v_0^2) = -q\Delta V$$
Charged Particle in a Uniform Field, Example

- A positive charge is released from rest and moves in the direction of the electric field.
- The change in potential is negative.
- The change in potential energy is negative.
- The force and acceleration are in the direction of the field.
- Conservation of Energy can be used to find its speed.
- This is useful HW 7-2 & 3

Section 25.2
7-2. An electron is placed half way between two parallel plates (A and B). Plate A is held at 0 V and plate B is held at 100 V. The electron will:

1. Hit plate A with 0 J of energy.
2. Hit plate B with 0 J of energy.
3. Hit plate A with $8 \times 10^{-18}$ J of energy.
4. Hit plate B with $8 \times 10^{-18}$ J of energy.
5. Hit plate A with $1.6 \times 10^{-17}$ J of energy.
6. Hit plate B with $1.6 \times 10^{-17}$ J of energy.
7-3. An electron is released from rest in a uniform electric field of magnitude

\[ E = [02] \text{ V/m}. \]  
(a) Through what potential difference will it have passed after moving 1.24 cm?  
(b) How fast will the electron be moving after having traveled that 1.24 cm?  
\[ \text{[(a) 40.0, 90.0 V (b) } 4.00 \times 10^6, 6.00 \times 10^6 \text{ m/s]} \]
Equipotentials

• Point $B$ is at a lower potential than point $A$.
• Points $A$ and $C$ are at the same potential.
  – All points in a plane perpendicular to a uniform electric field are at the same electric potential.
• The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.

Section 25.2
Field lines are always perpendicular to potential surfaces.
Consider moving a positive test between the following points on the equipotential surfaces shown.

A. A-B
B. B-C
C. C-D
D. D-E

Electric field lines always point in the direction of decreasing electric potential.

Doing this exercise helps with HW 8-3

Which way does the field point? Thought question

Quiz: Which movement involves no work?
Quiz: Which movement requires us to do the most positive work?
Quiz: Which movement lowers the potential energy the most?
Sample Exam Question

A plot of the electric field is shown in the figure. The numbered points are simply designated locations (not charges). Of the indicated points, the one at the highest potential is number 5. Find the pair of points having the smallest potential difference. The sum of their numbers is . Find the pair with the largest potential difference. The sum of their numbers is .
Point-charge example: use E to obtain $V$. 

$$V(r) = V(r) - V(\infty) = -\int_{\infty}^{r} E \cdot ds$$

$$= -\int_{\infty}^{r} (E \hat{r}) \cdot (\hat{r} dR)$$

$$= -\int_{\infty}^{r} E dR = -\int_{\infty}^{r} \frac{kQ}{R^2} dR$$

$$= -kQ \int_{\infty}^{r} \frac{dR}{R^2}$$

$$= -kQ \left( -\frac{1}{R} \right)_{\infty}^{r}$$

$$= -kQ \left( -\frac{1}{r} + \frac{1}{\infty} \right)$$

$$= \frac{kQ}{r}$$

$$U(r) = U(r) - U(\infty) = qV(r) = \frac{kQq}{r}$$
Electric Potential of a Point Charge

- The electric potential in the plane around a single point charge is shown.
- The red line shows the $1/r$ nature of the potential.

The red-brown curve shows the $1/r$ nature of the electric potential as given by Equation 25.11.
Electric Potential with Multiple Charges

• The electric potential due to several point charges is the sum of the potentials due to each individual charge.
  – This is another example of the superposition principle.
  – The sum is the algebraic sum \( V = k_e \sum_i \frac{q_i}{r_i} \)

• \( V = 0 \) at \( r = \infty \)
Electric Potential of a Dipole

• The graph shows the potential (y-axis) of an electric dipole.
• The steep slope between the charges represents the strong electric field in this region.
Electric potential near point charges: exercise for later

Which way will the field point?
(a) $+x$  (b) $-x$  (c) $+y$  (d) $-y$

Which way will the force point?
(a) $+x$  (b) $-x$  (c) $+y$  (d) $-y$
7-4. A charge of \(+q\) is at the origin and a charge of \(-q\) is at \(x = 2.000\) m. (a) For what finite positive values of \(x\) is the electric potential zero? (b) If \(q = 1.50\) nC, what is the magnitude of the electric field at this point? [(a) 0.300, 0.700 m (b) 40, 120 N/C]
iClicker Quiz
(1) I have completed at least 50% of the reading and study-guide assignments associated with the lecture, as indicated on the course schedule.

A. True  B. False

Today paying attention to minus signs & Δ (changes) remains very important. Also path integrals

1. Work: Gravitational and Electrical Energy
2. Change in Potential energy and kinetic energy.
3. Potential vs. potential energy:
4. Uniform fields, point charges and dipoles.
   1. When can you get rid of the Δ?
5. Applications: Integrals in 3D to get delta V & derivatives to get E field. Contour maps.
   1. Conductors
6. Calendar: HW 8 due Saturday 8:45 a.m.
   1. Math Rev Make up ends Friday.
Special case: point charge

\[ F = -\nabla U \]
\[ U = -\int F \cdot ds \]
\[ F = qE \]
\[ E = \frac{kQq\hat{r}}{r^2} \]
\[ U = qV \]
\[ V = \frac{kQ}{r} \]
\[ E = -\nabla V \]
\[ V = -\int E \cdot ds \]
Force

\( F \)

\[ F = -\nabla U \]

Potential Energy

\( U \)

\[ U = -\int F \cdot ds \]

\( F = qE \)

\[ U = qV \]

\( E = \frac{F}{q} \)

\[ V = \frac{U}{q} \]

\[ E = -\nabla V \]

\[ V = -\int E \cdot ds \]

Field

\( E \)

Potential

\( V \)
**Gradient:** a three-dimensional derivative
(three derivatives instead of one)

\[ \nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k} \]

A gradient always points in the direction of steepest ascent.

\[ \mathbf{E} = -\nabla V = -\left( \frac{\partial V}{\partial x} \mathbf{i} + \frac{\partial V}{\partial y} \mathbf{j} + \frac{\partial V}{\partial z} \mathbf{k} \right) \]

An \( \mathbf{E} \)-field always points in the direction of steepest descent.
Example of a potential gradient calculation

\[ V(x, y, z) = 2x^2 y - z \]

\[ \mathbf{E}_{(1,-1,0)} = -\nabla V = ? \]

\[ E_x = -\frac{\partial V}{\partial x} = -4xy \bigg|_{(1,-1,0)} = 4 \]

\[ E_y = -\frac{\partial V}{\partial y} = -2x^2 \bigg|_{(1,-1,0)} = -2 \]

\[ E_z = -\frac{\partial V}{\partial z} = 1 \bigg|_{(1,-1,0)} = 1 \]

\[ \mathbf{E} = 4\hat{i} - 2\hat{j} + \hat{k} \text{ ; at (1, -1, 0)} \]
Point-charge example: use $V$ to obtain $\mathbf{E}$.

$$\mathbf{E} = -\vec{\nabla} V = -\left( \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right) \quad V = \frac{kQ}{r}$$

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x} \left( \frac{kQ}{r} \right) = kQ \frac{\partial (1/r)}{\partial x}$$

$$\frac{\partial (1/r)}{\partial x} = \frac{\partial}{\partial x} \left( x^2 + y^2 + z^2 \right)^{-\frac{1}{2}} = -\frac{1}{2} \left( x^2 + y^2 + z^2 \right)^{-\frac{3}{2}} (2x) = -\frac{x}{r^3}$$

$$\mathbf{E} = -kQ \left( -\frac{x\hat{i} - y\hat{j} - z\hat{k}}{r^3} \right) = -kQ \left( -\frac{\mathbf{r}}{r^3} \right) = \frac{kQ\mathbf{r}}{r^3}$$

Suppose someone gave you a potential with terms like $9\ V[(x-1)^2 + [(y+2)^2 + [(z)^2]^{-\frac{1}{2}}$; could you give the magnitude, sign and position of the charge(s)? Could you calculate the electric field from this?
Potential from a conducting sphere with charge $Q$ on the surface

$$\mathbf{E}(\mathbf{r}) = \frac{kQ \hat{\mathbf{r}}}{r^2} \quad \text{if} \quad r \geq a \quad \text{or} \quad 0 \quad \text{if} \quad r < a$$

For $r \geq a$:

$$V(r) = V(r) - V(\infty) = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{s} = -\int_{\infty}^{r} (E \hat{\mathbf{r}}) \cdot (\hat{\mathbf{r}} dR)$$

$$\Rightarrow V(r) = \int_{r}^{\infty} \frac{kQ}{R^2} dR = \frac{kQ}{r}$$

For $r < a$,

$$V(r) = V(a) = \frac{kQ}{a}$$
Potential: positively-charged conducting shell, or an insulating shell with uniform surface charge density.
8-1. A hollow spherical metallic shell of radius of $R = 25$ cm holds a net surface charge of $Q = [01]$ _______ pC. (a) Calculate the electric potential at a distance of $2R$ from the center of the sphere. (b) Calculate the electric potential at the surface of the sphere. (c) Calculate the electric potential at the center of the sphere. 

[(a) $-1.00$, 1.00 V  
(b) $-1.00$, 1.00 V  
(c) $-1.00$, 1.00 V]
Constant Electric field: find potential at various places.

\[ \mathbf{E} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} \]

\[ V(1,1,1) = 10 \text{ Volts} \]

Find \( V(1,0,1) \)

\[ \Delta V = V(1,0,1) - V(1,1,1) = -\int_{(1,1,1)}^{(1,0,1)} \mathbf{E} \cdot d\mathbf{s} \]

\[ = -\mathbf{E} \cdot \int_{(1,1,1)}^{(1,0,1)} d\mathbf{s} = -\mathbf{E} \cdot \Delta\mathbf{s} \]

\[ = -\mathbf{E} \cdot [(1,0,1) - (1,1,1)] = -\mathbf{E} \cdot (0,-1,0) \]

\[ = -(2,-2,1) \cdot (0,-1,0) = -2 \]

\[ V(1,0,1) = V(1,1,1) + \Delta V \]

\[ = 10 - 2 = 9 \text{ Volts} \]
8-2. In the presence of a uniform electric field, \( \mathbf{E} = -(6.9 \text{ N/C})\mathbf{i} + (0.8 \text{ N/C})\mathbf{j} + E_z \mathbf{k} \), where \( E_z = [02] \) \( \text{N/C} \), assume that the electric potential is \(-20.00 \text{ V}\) at the origin of the coordinate system, and determine the electric potential at the point \( 2.7 \mathbf{i} - 2.7 \mathbf{k} \).
\([-15.00, 15.00 \text{ V}]\)
Nonconstant Electric field: find potential at various places.

\[ \mathbf{E} = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (x + y)\mathbf{k} \]

\[ V(0,0,0) = 0 \text{ Volts} \]

Find \( V(1,1,1) \)

\[ \Delta V = V(1,1,1) - V(0,0,0) \]

\[ = -\int_{(0,0,0)}^{(1,1,1)} \mathbf{E} \cdot ds \]

\[ = -\int_{(0,0,0)}^{(1,1,1)} (E_x, E_y, E_z) \cdot (dx, dy, dz) \]

\[ = -\left( \int_0^1 E_x \, dx + \int_0^1 E_y \, dy + \int_0^1 E_z \, dz \right) \]

\[ = -\left( \int_0^1 (0) \, dx + \int_0^1 (1) \, dy + \int_0^1 (2) \, dz \right) \]

\[ = -(0 + 1 + 2) = -3 \]

\[ V(1,1,1) = V(0,0,0) + \Delta V \]

\[ = 0 - 3 = -3 \text{ Volts} \]
iClicker Quiz
I have completed at least 50% of the reading and study-guide assignments of Chapters 23-5

A. True     B. False
Today paying attention to minus signs & Δ (changes) remains very important. Also path integrals
1. Work: Gravitational and Electrical Energy
2. Change in Potential energy and kinetic energy.
3. Potential vs. potential energy:
4. Uniform fields, point charges and dipoles.
   1. When can you get rid of the Δ?
5. Applications: Integrals in 3D to get delta V & derivatives to get E field.
   1. Contour maps.  2. Conductors 3. Potentials relative to objects.  4. Potential energy of charge distributions.  5. path integrals made EZ.
6. Calendar:  HW 9 due Tuesday 8:45 a.m.
   1. Exam 2 runs through Wednesday.
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Neither zero nor infinity are convenient zero-voltage references. HW 9 has a ln in the solution.
Review & more cases

Question: what best describes your familiarity with potential problems in set 9 ppt
A. I am fine go on
B. I have done them but could use some discussion.
C. I need discussion
Field lines and equipotential surfaces for a few simple configurations: uniform, monopole, and dipole fields.

(a) Uniform field
(b) Monopole field
(c) Dipole field

Equipotential surfaces are perpendicular to the field lines at every point, and densely spaced when the field lines are densely spaced. They are like elevation contours on a topographical map – it marks a region of constant voltage (height).
Quiz:
Where is the electric potential greatest?
\[ E(\mathbf{r}) = k \frac{3(\mathbf{p} \cdot \hat{r})\hat{r} - \mathbf{p}}{r^3} \]

\[ V(r) = ? \]

\[ = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{s} \]

\[ = -\int_{\infty}^{r} \left( k \frac{3(\mathbf{p} \cdot \hat{r})\hat{r} - \mathbf{p}}{R^3} \right) \cdot (\hat{r} \, dR) \]

\[ = -2k \mathbf{p} \cos(\theta) \int_{\infty}^{r} \frac{dR}{R^3} \]

\[ = \frac{k \mathbf{p} \cos(\theta)}{r^2} \]
Despite a complicated surface charge density, the entire surface of the conducting sphere has the same potential.
In electrostatic equilibrium, conducting objects are equipotential bodies, and therefore have equipotential surfaces.
Insulating ring of radius $a$ and linear charge density $\lambda$

See Example 25.5 p. 704 7th edition

\[ V = \int dV \]

\[ dV = \frac{kdQ}{r} = \frac{k\lambda ds}{r} \]

\[ = \frac{k\lambda(a d\phi)}{a} = k\lambda d\phi \]

9-2. A uniformly charged insulating rod of length 60.0 cm is bent into the shape of a semicircle. If the rod has a total charge of $Q = [02] \, \frac{26.9}{\,\text{pC}}$. Find the electric potential at the center of the semicircle. $[\,2.50, \, 2.50 \, \text{V}]$ \, \text{1.27 V}
9-1. A uniformly-charged rod of length $L = 2.00 \, \text{m}$ and charge density $\lambda = 2.65 \times 10^{-9} \, \text{C/m}$ lies along the x axis with its left end at the origin. Calculate the electric potential at the point located a distance $d = \boxed{0.88} \, \text{m}$ beyond the end of the rod along the $-x$ axis. [10.0, 50.0 \, \text{V}]

Answer: 28.2 \, \text{m}

\[ V = \int dV = k_E \lambda \int \frac{dx}{x}; \text{ where the limits of integration are } d \text{ to } d+2.00 = k_E \lambda \ln \left(\frac{(d+2.00)}{2.00}\right) \]
Insulating annulus with uniform surface charge density $\sigma$, inner radius $a$ and outer radius $b$

\[
V = \int dV
\]

\[
dV = \frac{k dQ}{r} = \frac{k \sigma dA}{r}
\]

\[
= \frac{k \sigma (2\pi s ds)}{\sqrt{s^2 + z^2}}
\]

Example 25.6 p. 705
9-3. Calculate the electric potential at a point \( x = 0.489 \text{ m} \) along the axis of the annulus as shown. The annulus has a uniform charge density of \( \sigma = 1.35 \text{ \( \mu \)C/m}^2 \), an outer radius of \( b = 1.13 \text{ m} \) and an inner radius of \( a = \boxed{0.19} \text{ m} \). [20.0, 60.0 kV]

\[ 53.9 \text{ kV} \quad \text{Go to Review.} \]
Potential from a conducting sphere with charge $Q$ on the surface

$$V(\mathbf{r}) = \int dV(\mathbf{r}) = \int \frac{k dQ}{s} = -k \sigma a \int_{0}^{2\pi} \int_{1}^{1} \frac{1}{(1 + \xi^2 - 2\xi \kappa)^{1/2}} d\kappa d\phi$$

$$= 2\pi k \sigma a \left( |\xi + 1| - |\xi - 1| \right) \xi$$

$$\mathbf{r} = r \hat{\mathbf{z}}$$

$$\mathbf{a} = a \hat{\mathbf{r}}$$

$$\mathbf{s} = \mathbf{r} - \mathbf{a}$$

$$\xi \equiv \frac{r}{a}$$

$$\sigma = \frac{Q}{4\pi a^2}$$

$$dQ = \sigma dA = \sigma a^2 \sin(\theta) d\theta d\phi$$

$$\kappa \equiv \cos(\theta) \quad d\kappa \equiv -\sin(\theta) d\theta$$

$$s = |\mathbf{r} - \mathbf{a}| = \sqrt{r^2 + a^2 - 2ra \cos(\theta)} = a \sqrt{1 + \xi^2 - 2\xi \kappa}$$

$$V(r) = \frac{kQ}{r} \quad \text{if} \quad r \geq a \quad \text{or} \quad \frac{kQ}{a} \quad \text{if} \quad r \leq a$$
Point charge potential
Potential: positively-charged solid conducting sphere
Potential: positively-charged conducting shell, or an insulating shell with uniform surface charge density.
Potential: positively-charged solid insulating sphere
Positive point charge within a thick neutral conducting shell
Negative point charge within a thick neutral conducting shell