Kinetic theory

class 15:
(ThT Q)
Did you complete at least 70% of Chapter 21:1-3?

A. Yes

B. No

We will use the CS Review
KT: using online demo. & formulae
Degrees of freedom
Heat capacity ratio:

\[ \gamma = \frac{C_p}{C_v} \] & other

\[ C_P-C_V \] relations
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Remember: the online schedule has precedence.

Today:
1. Kinetic theory of gases.
2. Degrees of Freedom
3. \( \gamma \), the adiabatic exponent

Wed noon

#2 #3 begin
Consider a mole of He at 300K. If we **compress** the gas to half its volume, which process requires the most work? pp.

A. isothermal
B. adiabatic
C. both same

Note: lines go backward in compression; but from where do you start?
Figure 21.5

Isotherms

Adiabatic process

$P_i$, $T_i$, $P_f$, $T_f$, $V_i$, $V_f$
Microscopic view of how atoms hitting wall creates pressure.
The ideal gas law works for all atoms and molecules at low pressure. It is rather amazing that it does. Kinetic theory explains why. The properties of an ideal gas can be understood by thinking of it as \(N\) rapidly moving particles of mass \(m\). As these particles collide with the container walls, momentum is imparted to the walls, which we call the force of gas pressure. In this picture the pressure is related to the average of the square of the particle velocity by

\[
P = \frac{2}{3} \left( \frac{N}{V} \right) \left( \frac{1}{2} m \bar{v}^2 \right)
\]
Average translational kinetic energy

$$\left( \frac{1}{2} m v^2 \right)_{avg} = \frac{3}{2} K_b T$$

$$K_B = \frac{R}{N_A}$$

Boltzmann’s Constant

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$
Amazing Results!

- $<KE>$ depends only on $T$! Now, we can call the total kinetic energy of the gas the thermal energy of the gas. For an ideal gas, this is the internal energy, $E_{\text{int}}$. So $E_{\text{int}}$ is proportional to $T$.

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

- Air at $20^\circ\text{C}$ $\implies V_{\text{rms}} = 500 \text{ m/s} = 1100 \text{ mph}$
Which molecule has the highest average velocity?

In our classroom there are molecules of $\text{N}_2$ and $\text{CO}_2$. $\text{N}_2$ has less mass than $\text{CO}_2$ ($M=28$ vs. $44$). Which molecule has the highest average velocity?

A. Nitrogen
B. Carbon Dioxide
C. You can’t trick us, Prof. Allred, they are the same.
Formulae

1. \[ P = \frac{2}{3} \left( \frac{N}{V} \right) \left( \frac{1}{2} m \overline{v^2} \right) \]

2. Using the ideal gas law, we obtain the average translational kinetic energy per molecule:

\[ \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T \]

3. The rms speed is then given by

\[ v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3 k_B T}{m}} = \sqrt{\frac{3 R T}{M}} \]
Degrees of Freedom:

Roughly speaking, a degree of freedom is a way in which a molecule can store energy. For instance, since there are three different directions in space along which a molecule can move,

1. there are three degrees of freedom for the translational kinetic energy.

2. There are also three different axes of rotation about which a polyatomic molecule can spin, so we say there are three degrees of freedom for the rotational kinetic energy.

3. There are even degrees of freedom associated with the various ways in which a molecule can vibrate, and with the different energy levels in which the electrons of the molecule can exist.
It takes more Q to raise temperature at constant P than constant V, because some Q goes to work.

Higher temperatures are the upper blue lines. For ideal gases the blue lines also mark the lines of constant $E_{\text{int}}$. 
Molar Specific Heat of an Ideal Gas at Constant Volume:

\[ Q = nC_V \Delta T \quad C_V > \frac{5}{2} \] (polyatomic)

\[ C_V = \frac{3}{2} R \] (monatomic) \quad \[ C_V = \frac{5}{2} R \] (diatomic)

Real gases deviate from these formulas because in addition to the translational and rotational degrees of freedom, they also have vibrational and electronic degrees of freedom. These are unimportant at low temperatures due to quantum mechanical effects, but become increasingly important at higher temperatures. The rough rule is:

\[ C_V = \frac{\text{No. of degrees of freedom}}{2} R \]
Consider a mole of He in a sealed jar and a mole of N₂ in another. We heat each gas to raise its temperature from 20° C to 100° C. For which gas will the $\Delta E_{\text{int}}$ be greatest?

A. He
B. N₂
C. both same
Molar Specific Heat of an Ideal Gas at Constant Pressure:

The internal energy of an ideal gas depends only on the temperature:

\[ Q = nC_P \Delta T \]
\[ C_P = C_V + R \]

The internal energy of an ideal gas depends only on the temperature:

\[ E_{\text{int}} = nC_V T \]
Consider a mole of $\text{N}_2$. We heat the gas to raise its temperature from $20^\circ \text{C}$ to $100^\circ \text{C}$. For which case will the $\Delta E_{\text{int}}$ be greatest?

A. Constant volume
B. Constant Pressure
C. both same
Figure 20.8

A process path labeled A, B, C, and D moves through different temperature levels labeled $T_1$, $T_2$, $T_3$, and $T_4$. The path starts at A and ends at D, indicating a change in state from one temperature level to another.
Adiabatic Processes in an Ideal Gas: $\gamma = \frac{C_P}{C_V}$.

An adiabatic process is one in which no heat is exchanged between the system and the environment. When an ideal gas expands or contracts adiabatically, not only does its pressure change, as expected from the ideal gas law, but its temperature changes as well. Under these conditions the final pressure, $P_f$, can be computed from the initial pressure, $P_i$, and from the final and initial volumes, $V_f$ and $V_i$, by

$$P_f V_f^\gamma = P_i V_i^\gamma \quad \text{or} \quad PV^\gamma = \text{constant}$$

$\gamma = \frac{C_P}{C_V}$. 
The quantity $\gamma$ is called the \textit{adiabatic exponent.}

For monatomic gases: the noble gases: He, Ar, Ne etc. $\gamma = \frac{C_p}{C_v} = \frac{5/2R}{3R/2} = \frac{5}{3} = 1.67$

diatomics like N$_2$ and O$_2$; $\gamma = \frac{7/2R}{5R/2} = 1.4$

Note that this doesn’t mean that the ideal gas law no longer holds; it does, and in fact it can be combined with the adiabatic law for pressure given above to obtain the adiabatic law for temperatures: This is useful for computing T.

$$TV^{\gamma^{-1}} = \text{constant}$$
Consider a mole of He at 300K. If we **compress** the gas to half its volume, which process requires **the most work**? *pp.*

1. isothermal
2. adiabatic
3. both same

Note: lines go backward in compression; but from where do you start?
• 1 mol He at 300 K
  – Compress to $\frac{1}{2}$ volume isothermally

$$W = nRT \ln \frac{V_f}{V_i} = 1730J$$
• 1 mol He at 300K
  – Compress to ½ volume adiabatically

• Eq \_1 (21,20) in textbook: \( T_i V_i^{\gamma^{-1}} = T_f V_f^{\gamma^{-1}} \)

\[ \Rightarrow T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma^{-1}} = 476 K \]

\[ \Delta E_{\text{int}} = Q + W = W \]

\[ \Rightarrow W = \Delta E_{\text{int}} = nC_v\Delta T = \frac{3}{2}nR\Delta T \]

\[ = 2194 J \]

Here is why the adiabatic process takes more work. The work going in raises the temperature, this increases the pressure which makes it more difficult to compress the gas.
Consider a mole of He gas and a mole of N₂ gas, both at the same initial pressure and temperature. Which requires the most work to adiabatically compress to half its volume?

A. He
B. N₂
C. Both the same
• Work → Internal energy (kinetic energy)
• He: all W → translational K
• N₂: W ► translational K
  + rotational K
  ► energy from work spreads out.
  ► molecules don’t go as fast.
  ► temperature doesn’t rise as far.
  ► pressure doesn’t rise as much.
  ► less pressure ► less work.
Compressions (& rarefactions) in sound waves are adiabatic because they happen too rapidly for any appreciable amount of heat to flow. This is why the adiabatic exponent, $\gamma$, appears in the formula for the speed of sound in an ideal gas:

$$v = \sqrt{\frac{\gamma RT}{M}}$$
Sound Waves

Compressions are adiabatic

\[ V = \sqrt{\frac{\gamma RT}{M}} \]

**Air:** \( \gamma = 1.40; \) \( M=29 \text{ g/mol} \) \( v = 343 \text{ m/s at } 20^\circ \text{C} \)

**He:** Larger \( \gamma \) ➔ Larger \( v \) & Smaller \( M \) ➔ Larger \( v \)
So \( v \) in He is very large!

Standing wave in a tube: \( f= \frac{v}{2L}; \) if \( v \uparrow \) then \( f \uparrow \)
(Squeaky voice and flute note frequency \( \uparrow \).)

**Computational Hint:** when computing \( v \) using \( M \), remember to convert \( M \) from grams to kg.

\[ F = \frac{V}{2L} \]
13-1 -3 are based on this material. #2 have the heat capacity of Cu (review) and heating of gasses

13.4 has work involved. We will do a sample numerical problem.
We heat up 2.43 mol of some gas from 21.7°C to [05]°C, holding the pressure constant. The molar specific heat at constant volume for this gas is CV = [06] J/mol · K.

(a) Assuming that this gas behaves like an ideal gas, find the value of the molar specific heat at constant pressure CP.

(b) Find the amount of heat that flows into the gas.

(c) Find the amount of work done on the gas.

(d) Find the change of internal energy of the gas.