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Testing two crackle criteria using modified jet noise waveforms

S. Hales Swift,[a] Kent L. Gee, and Tracianne B. Neilsen
Department of Physics and Astronomy, Brigham Young University, Provo, Utah 84602, USA
hales.swift@gmail.com, kentgee@byu.edu, tbn@byu.edu

Abstract: The jet sound quality "crackle" has historically been studied and quantified using the statistics of the pressure waveform. Some investigators have suggested crackle, and its associated shock content, may be better quantified using the statistics of the time derivative of the waveform. Modified waveforms are used to evaluate crackle prediction criteria based on the skewness of each variable. The resultant waveforms are provided as direct evidence that the pressure distribution does not directly predict or quantify a crackling sound quality while the statistics of the derivative appear to do so.

[SKL]
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1. Introduction

Ffowcs Williams *et al.* (1975) first described crackle in what have become [in company with Richardson's (1922) "whorls" poem] some of the most oft-quoted and memorable lines in aeroacoustics, "Some observers liken it to the sound of an electric arc welder or of a badly connected loud speaker: others liken it to the spitting of water added to extremely hot fat. It is a startling staccato of cracks and bangs and its onomatope, 'crackle,' conveys a subjectively accurate impression." This sound quality—interesting to Ffowcs Williams because of its capacity to produce annoyance—cannot be identified by examination of the long-term spectral magnitude alone or any metric derived therefrom, as shown by Gee *et al.* (2007b). It is instead "caused by groups of sharp compressions in association with gradual expansions" as noted by Ffowcs Williams *et al.* (1975). In an attempt to quantify the presence of the spectrally elusive crackle, Ffowcs Williams *et al.* suggested the use of the pressure waveform skewness ($\mathrm{Sk}\{p\}$)—the normalized third central moment of the pressure time series p—as a metric, noting an apparent correspondence between elevated values of $\mathrm{Sk}\{p\}$ and the presence of a crackling sound quality. $\mathrm{Sk}\{p\} \geq 0.4$ was associated with observations of clear crackle, while sounds with $\mathrm{Sk}\{p\} \leq 0.3$ did not crackle. On the basis of this association, they concluded that $\mathrm{Sk}\{p\}$ quantified the crackle percept, and this assumed equivalence has informed a great deal of further research.

Ffowcs Williams' quantifier, $Sk\{p\}$, has been examined as an indicator of crackle in works by Krothapalli *et al.* (2000), Papamoschou and Debiasi (2001), Petitjean *et al.* (2006), Krothapalli *et al.* (2011), Mora *et al.* (2013), McInerny *et al.* (2006), and Buchta and Freund (2016). It is thus clear that Ffowcs Williams' criterion has been influential in many of the questions that have been asked and investigated and the measurements that have been pursued and performed.

However, as crackle research has matured, concerns have surfaced about the adequacy of the skewness of the pressure waveform as a predictor and quantifier of a crackling sound quality. Papamoschou and Debiasi (2001) noted the incompleteness of the pressure skewness as a descriptor of crackle on the basis of its failure to "capture the sharpness of the pressure waves, which is the source of the annoyance." Baars and Tinney (2014) likewise noted that a "shortcoming in [Ffowcs Williams'] criterion is that the rise times of the compressive parts of the waveform are not taken into account." Both of these critiques point to the importance of shock content in quantifying crackle.

Shock content can be evaluated more directly by considering the skewness of the time-derivative of the pressure waveform $\mathrm{Sk}\{\partial p/\partial t\}$. Reichman *et al.* (2016) identified a derivative skewness value of approximately $\mathrm{Sk}\{\partial p/\partial t\} \approx 5$ as a key boundary for identifying the presence of significant shock content. Gee *et al.* (2016) pointed out that

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[a] Author to whom correspondence should be addressed.

the spatial regions of peak pressure skewness and peak derivative skewness are not necessarily well-aligned (e.g., an F-35 measured at a 38 m arc showed a peak in $\mathrm{Sk}\{p\}$ near 117° from the inlet while $\mathrm{Sk}\{\partial p/\partial t\}$ peaked near 137°) and that, to the extent that significant shock content is important to the crackle phenomenon, such content was better quantified using $\mathrm{Sk}\{\partial p/\partial t\}$ rather than $\mathrm{Sk}\{p\}$. On the basis of the combination of these two ideas this paper compares the criterion of $\mathrm{Sk}\{\partial p/\partial t\} \gtrsim 5$ with the Ffowcs Williams crackle criterion, $\mathrm{Sk}\{p\} \geq 0.4$ as a predictor of crackling sound quality.

Most relevant to the present study, Gee *et al.* (2007a) have also challenged the adequacy of the Ffowcs Williams crackle criterion on perceptual grounds. They filtered an initially Gaussian noise waveform to match the spectrum of a crackling jet waveform. They then nonlinearly transformed the filtered waveform to match the pressure skewness of 0.6 from the crackling jet waveform. No crackling sound quality was identified in their informal listening test. This predictive failure of the Ffowcs Williams crackle criterion implied that pressure skewness was not a sufficient condition for a crackling sound quality. The question which naturally follows—whether pressure skewness is a necessary condition—is addressed in this study. The waveform modification experiments presented in this paper show—giving the resultant waveforms and their statistical properties as direct evidence—that the distribution of the pressure waveform (and Ffowcs Williams' criterion) neither predicts nor quantifies a crackling sound quality, while the distribution of the derivative (and the derivative-based criterion) appears to do so.

2. Experiments

In order to efficiently evaluate the ability of the two criteria to predict and quantify crackle, transformations are desired that selectively modify the waveform with respect to one criterion while leaving it invariant with respect to the other. Spectral changes should also be minimized so as to allay concerns that filtering is the source of any qualitative change. Ideally, the alteration of the pressure and derivative distributions (which are the direct target of the waveform modifications) should be isolated as causative agents of changes in sound quality. Detailed discussion of the transformations used in this paper and code for their implementation can be found in Swift *et al.* (2017).

In examining some of the spectra shown by Swift *et al.* (2017) wherein these methods were applied, it was noted that significant spectral changes had occurred. Ffowcs Williams noted the impossibility of identifying crackle from the spectrum alone. However, some spectral alterations, such as low-pass filtering, can affect the perception of crackle in a signal, and alterations that affect crackle can also affect the spectrum. To compensate for the spectral changes resulting from the transformations, a spectral mask is applied to modified waveforms before calculating statistics; all now have the same long-term spectral magnitude as the original waveform. This spectral mask alters the time-domain statistics somewhat from the initial transformation, but does not impact the study's conclusions.

2.1 Crackling jet waveform

Rather than beginning with an initial Gaussian noise waveform, as was done in the previous study by Gee *et al.* (2007a), this study begins with a recording of a crackling jet. Beginning with a crackling waveform enables an investigation into which types of modifications remove crackle. The initial waveform has pressure skewness $\mathrm{Sk}\{p(t)\} = 0.57$ and derivative skewness $\mathrm{Sk}\{\partial p/\partial t\} = 5.59$. The jet crackles audibly, as predicted by either criterion ($\mathrm{Sk}\{p(t)\} \geq 0.4$ or $\mathrm{Sk}\{\partial p/\partial t\} \gtrsim 5$) and, as suggested by $\mathrm{Sk}\{\partial p/\partial t\}$, has significant shock content. The amplitude- and timescaled waveform can be heard in Mm. 1. (Note: Please listen to these signals only at safe and comfortable levels.)

Mm. 1. Original (crackling) jet waveform. This is a file of type ".wav" (1025 kB).

2.2 Transformation of p or ∂p/∂t to exhibit a Gaussian distribution

A main goal of the modification strategy is to affect the statistics of $\mathrm{Sk}\{p(t)\}$ and $\mathrm{Sk}\{\partial p/\partial t\}$ independently. To that end, a custom nonlinear transformation, T, is constructed that maps values of a time series to values of a Gaussian, G, with the same standard deviation, while maintaining key temporal behaviors. To do this, each value $a(t)$ of a time series is mapped to its associated cumulative distribution function (CDF) value $\mathrm{CDF}(a(t)) \in [0,1]$ These values are then input into the inverse CDF of a truncated Gaussian distribution of the same standard deviation. The time series produced has similar features and the same standard deviation, but a Gaussian distribution. T preserves order in time and magnitude—if $a(t_1) > a(t_2)$ then $T\{a(t_1)\} > T\{a(t_2)\}$—and preserves the locations of maxima and minima. Because T is continuous, it preserves continuity, maps discontinuities to discontinuities and maintains boundedness.

This transformation technique can be applied to either the pressure time series ($T_p\{p\} \rightarrow G$), as in Mm. 2, leading to a removal of $\mathrm{Sk}\{p(t)\}$, or to the derivative time series ($T_d\{\partial p/\partial t\} \rightarrow G$) leading to a removal of $\mathrm{Sk}\{\partial p/\partial t\}$, as in Mm. 3. In the case where it is applied to the derivative it is necessary to reintegrate the waveform. Integration of the transformed derivative time series is performed using a leaky integration, as discussed in Eliasmith and Anderson (2004) [symbolized as $\int_L(\partial p/\partial t)dt$]. The transformation of the derivative is no longer zero-sum and if integrated in the normal manner this leads to significant departures from zero-mean behavior. Differentiation followed by leaky integration has a high-pass filtering effect, which counteracts the departure from zero-mean behavior that results from integrating the transformed signal. This process thus helps restore zero-mean behavior in the signal. To test its independent effects, leaky integration is also applied to the derivative of the original crackling jet waveform in Mm. 4. Because Mm. 4 still crackles, the leaky integration is not responsible for the removal of crackle in Mm. 3.

Mm. 2. Original jet noise waveform with p nonlinearly transformed to Gaussian PDF ($T_p\{p\}\rightarrow G$). This is a file of type ".wav" (1025 kB).

Mm. 3. Original jet noise waveform with $\partial p/\partial t$ nonlinearly transformed to Gaussian PDF ($T_d\{\partial p/\partial t\}\rightarrow G$). This is a file of type ".wav" (1025 kB).

Mm. 4. Original jet noise waveform differentiation and leaky-integration ($\int_L(\partial p/\partial t)dt$). This is a file of type ".wav" (1025 kB).

2.3 "Slowing" shocks

Finally, a modification aimed at slowing the shocks adds points to increase their rise times. Additional points are linearly interpolated between pairs of adjacent pressure time series values that differ by more than a chosen threshold so that the rate of change post-interpolation is less than the chosen threshold. This slows the rise rate of shocks to at or below a given "speed limit." The speed limit chosen was a change between subsequent values of the pressure waveform of 30.42% of the standard deviation of the time waveform. The resultant sound is Mm. 5.

Mm. 5. Original jet noise waveform modified by slowing the rise time of shocks to at or below a chosen rate. This is a file of type ".wav" (1025 kB).

3. Results

All three transformation types were applied separately to copies of the crackling jet waveform. Probability density functions (PDF) of p and $\partial p/\partial t$ (first difference) and spectra were calculated for each modified waveform, and the presence of a crackling sound quality was evaluated using informal listening tests. Results detailing the PDF and spectral effects of the transformations are given in Swift *et al.* (2017); however, in the results of Swift *et al.* (2017) the spectral mask was not imposed. The results shown in this paper include the spectral mask unless otherwise noted. The power spectra post-mask are shown in Fig. 1(a) where the use of the spectral mask results in nearly perfect agreement between the spectra of the modified waveforms.

Independent of spectral consequences, the PDFs of p and $\partial p/\partial t$ in Figs. 1(b) and 1(c) indicate the changes effected by each alteration strategy. In both PDF plots a Gaussian distribution with the mean and standard deviation of the original waveform is included in gray for a visual reference. Differences in $\mathrm{PDF}\{p\}$ can be seen in Fig. 1(b). The deviations from Gaussian behavior in $\mathrm{PDF}\{p\}$ are relatively small. Larger differences are seen in $\mathrm{PDF}\{\partial p/\partial t\}$. The horizontal axis of $\mathrm{PDF}\{\partial p/\partial t\}$ is the normalized pressure difference between successive pressure time series values or "first difference." The first difference is used as a proxy for $\partial p/\partial t$ throughout this work. Of particular significance in the plot of $\mathrm{PDF}\{\partial p/\partial t\}$ is the "positive arm" seen for three of the waveforms (black, green and blue)—the original crackling waveform, the leaky integration of the derivative of the original waveform [$\int_L(\partial p/\partial t)dt$] and the waveform from the transformation of the original to a Gaussian distribution ($T_p\{p\}\rightarrow G$). Each of these waveforms exhibits crackle, and each has a virtually identical derivative distribution. Neither of the waveforms (cyan and red) in this study that lack this "arm" crackle. A similar arm-type feature is also visible after application of T_d to the derivative (cyan) and results from the spectral mask; however, it is of much lower magnitude given the log scale of the plot's vertical axis, with deviation from essentially Gaussian behavior occurring only at 2 orders of magnitude down from the peak.

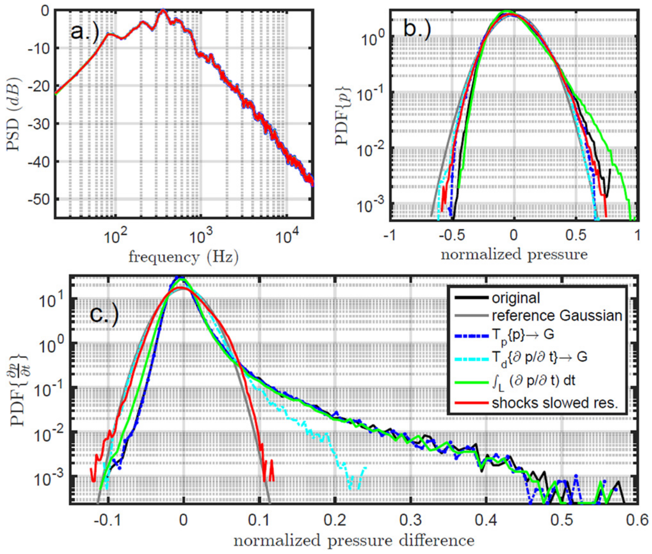

Fig. 1. (Color online) (a) Spectra of all signals. (b) PDFs of p for all signals. (c) PDFs of $\partial p/\partial t$ for all signals. The common legend in (c) applies to all subparts of this figure.

Skewness statistics used to evaluate the two crackle criteria are summarized in Table 1. Ffowcs Williams' criterion for crackle is $\mathrm{Sk}\{p\} > 0.4 \rightarrow$ crackles distinctly, $\mathrm{Sk}\{p\} < 0.3 \rightarrow$ no observable crackle. The derivative-based crackle criterion is $\mathrm{Sk}\{\partial p/\partial t\} \gtrsim 5 \rightarrow$ crackles distinctly. Informal listening test results gauging the presence of crackle in each signal are reported in the table.

As shown in Table 1, two of the transformations eliminated crackle, and three of them did not. Neither transformation of the pressure time series to a Gaussian distribution ($T_p\{p\}\rightarrow G$) nor differentiation followed by leaky integration of the signal [$\int_L(\partial p/\partial t)dt$] led to crackle removal, though both significantly altered the distribution of the pressure waveform. Transformation of the pressure waveform to a Gaussian distribution reduced skewness values to nearly zero, though application of the spectral mask increased the pressure skewness value to 0.27—still less than Ffowcs Williams' cut-off for non-crackling sounds. Differentiation followed by leaky integration led to a significantly increased pressure skewness value of 0.94, well above Ffowcs Williams' $\mathrm{Sk}\{p\} \geq 0.4$ criterion. Neither transformation in pressure (Mm. 2) nor differentiation followed by leaky integration of the original signal (Mm. 4) led to a significant change in crackle. Both resultant waveforms exhibit audible crackle to approximately the same degree as the original crackling jet waveform (Mm. 1). Although it did not affect the crackling sound quality, applying the spectral mask to obtain Mm. 2 led to an audible increase in low frequencies (relative to the sound's pre-mask quality).

Two alterations led to crackle removal: transforming $\partial p/\partial t$ to exhibit a Gaussian distribution ($T_d\{\partial p/\partial t\}\rightarrow G$), and slowing (increasing the rise time) of shocks. Transforming $\partial p/\partial t$ to a Gaussian distribution with leaky integration directly reduced $\mathrm{Sk}\{\partial p/\partial t\}$, thus the derivative-based criterion predicted no crackle. However, because $\mathrm{Sk}\{p\}$ also decreased, Ffowcs Williams' criterion also predicted no crackle.

Slowing shocks led to only minute changes in $\mathrm{Sk}\{p\}$, but a significant change in $\mathrm{Sk}\{\partial p/\partial t\}$. Because $\mathrm{Sk}\{\partial p/\partial t\}$ is directly reduced by shock slowing, the derivative-based criterion predicts no crackle. Prior to the application of the spectral mask,

Table 1. Skewness statistics and informal subjective crackle assessments for each waveform are given. Statistical values that satisfy a given crackle criteria are printed in **bold**. The links associated with each waveform are included.

Waveform	original	$T_p\{p\}\rightarrow G$	$T_d\{\partial p/\partial t\}\rightarrow G\rightarrow\int_L\left(\frac{\partial p}{\partial t}\right)dt$	$\int_L\left(\frac{\partial p}{\partial t}\right)dt$	Shock slow. res.
Crackle?	**Y**	**Y**	N	**Y**	N
Link:	Mm. 1	Mm. 2	Mm. 3	Mm. 4	Mm. 5
$\mathrm{Sk}\{p\}$	**0.57**	0.27	0.15	**0.94**	0.30,[a] **0.56**[b]
$\mathrm{Sk}\{\partial p/\partial t\}$	**5.59**	**5.40**	0.56	**4.95**	0.29

[a] Post-mask values of the slow shock signal.

[b] Pre-mask value of the slow shock signal.

Table 2. Crackling (bold) or non-crackling (regular) sound quality organized by skewness magnitude of the pressure and derivative time-series.

	$\mathrm{Sk}\{p\} > 0.4$	$\mathrm{Sk}\{p\} < 0.3$
$\mathrm{Sk}\{dp/dt\} \gtrsim 5$	**Original** $\boldsymbol{\int_L \left(\frac{\partial p}{\partial t}\right) dt}$	$\boldsymbol{T\{p\} \rightarrow G}$
$\mathrm{Sk}\{dp/dt\} < 5$	Slowed shocks[a] Gee *et al.* (2007a) $T\{G\} \rightarrow \mathrm{Sk}$	Slowed shocks[b] $T\{dp/dt\} \rightarrow G \Rightarrow \int_L \left(\frac{\partial p}{\partial t}\right) dt$

[a] Pre-mask values of the slow shock signal.

[b] Post-mask value of the slow shock signal.

Ffowcs Williams' criterion predicts crackle. Applying the spectral mask decreased $\mathrm{Sk}\{p\}$ enough that Ffowcs Williams criterion predicts no crackle, but the crackling sound quality of the sound was unchanged by the mask. Because employing the spectral mask did not reverse crackle removal, the signal is shown in Table 1 and Table 2 with both its pre-mask value ($\mathrm{Sk}\{p\} = 0.56$, marked with [b]) and its post-mask value ($\mathrm{Sk}\{p\} = 0.30$, marked with [a]). The mask did affect the sound quality, leading to increased smooth high-frequency noise rather than crackle. This suggests that the crackle removal was not due to a filter-like effect.

4. Concluding discussion

Having considered the direct results of the experiments carried out in Sec. 3, their implications for quantifying the crackle percept are now considered. To illustrate these implications, the waveforms are organized with respect to their categorization under the Ffowcs Williams and derivative-based crackle criteria in Table 2. Crackling sounds are indicated using bold text.

While Gee *et al.* (2007a) showed that $\mathrm{Sk}\{p\} \geq 0.4$ was not a sufficient condition for crackle perception, the present experiment has shown that it is also not a necessary condition. By transforming $p(t)$ to have a Gaussian distribution without removing crackle the pressure skewness is clearly shown to be unnecessary to the perception of crackle. The tabulated examples of false positive and false negative predictions using the $\mathrm{Sk}\{p\}$-based Ffowcs Williams criterion suggests a severe limitation in its predictive capacity. In contrast, the $\mathrm{Sk}\{\partial p/\partial t\}$-based criterion successfully predicts crackling or non-crackling sound quality in all of these signals. Considering the differences in spatial patterns of $\mathrm{Sk}\{p\}$ and $\mathrm{Sk}\{\partial p/\partial t\}$ that exist in the vicinity of high-performance aircraft noted in Gee *et al.* (2016), the disadvantage of using $\mathrm{Sk}\{p\}$ as a proxy for crackle becomes apparent.

Shifting entirely to the perceptual domain, sound quality metrics such as time-varying loudness, sharpness and roughness, and metrics derived therefrom should be investigated as possible means of quantifying crackling sound quality as suggested by Swift and Gee *et al.* (2011). Additionally, the derivative-based criterion under consideration here should be further developed to take into consideration marginal cases where a degree of crackle is observed but the feature is not present in full prominence. Values and conditions should be determined under which crackle is absent, audible, distinct, dominant and so forth. The waveform modifications used in this investigation could provide stimuli for such a study.

Finally, it seems that it has become necessary to separate the terms used for the description of a crackling sound quality and the phenomenon that leads to skewed pressure distributions in the near-field of jets. Both are interesting and worthy pursuits, but neither is well served by conflating the two phenomena.

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