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Nonlinearity Analysis of Model-scale Jet Noise

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Abstract. This paper describes the use of a spectrally-based "nonlinearity indicator" to complement ordinary spectral analysis of jet noise propagation data. The indicator, which involves the cross spectrum between the temporal acoustic pressure and the square of the acoustic pressure, stems directly from ensemble averaging the generalized Burgers equation. The indicator is applied to unheated model-scale jet noise from subsonic and supersonic nozzles. The results demonstrate how the indicator can be used to interpret the evolution of power spectra in the transition from the geometric near to far field. Geometric near-field and nonlinear effects can be distinguished from one another, thus lending additional physical insight into the propagation.

Keywords: nonlinear, propagation, jet, noise, supersonic, Burgers equation
PACS: 43.50.Nm, 43.25.Cb, 43.60.Wy

INTRODUCTION

In the analysis of high-amplitude jet noise for nonlinear acoustic propagation, limitations in measurement bandwidth and propagation range can create difficulties in identifying nonlinear propagation effects via comparisons of power spectra measured at different distances. This can be of particular importance in anechoic, model-scale jet noise measurements, where many frequencies of interest are above the audio range and maximum propagation range is limited by chamber size. Limitations in measurement bandwidth or range may lead to erroneous conclusions regarding the nonlinearity of the propagation made using the spectral comparison method can be reached. Hence, the ability to extract evidence of nonlinearity directly from time waveform analysis can be important.

This paper describes nonlinearity analyses of data collected largely in the geometric near field of a laboratory-scale, ideally expanded, Mach-2.0, unheated jet. As such, comparisons of power spectra along radials extending out from the centerline do not collapse. However, the potential causes for this failure to collapse, e.g., linear propagation from extended, directional sources, or nonlinear propagation, cannot be distinguished. Although a previous paper investigated application of the bicoherence,[1] this paper utilizes analysis techniques based on the generalized Burgers equation.[2] Additional details regarding the experiment may be found in Refs. 1 and 3.

NONLINEAR ACOUSTICS State-of-the-Art and Perspectives
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QUADSPECTRAL ANALYSIS

In spectral analysis, the quadspectral density is defined as the imaginary part of the cross spectral density between two signals. The particular quantity that is useful as an indicator of nonlinear propagation is the quadspectral density between the square of the acoustic pressure and the acoustic pressure, namely

$$Q_{p^2p}(f) = \lim_{T\rightarrow\infty} \frac{1}{T} \langle P_{sq}(f) P^*(f) \rangle. \tag{1}$$

Morfey and Howell[3] first recognized the potential of Q_{p^2p} as a nonlinearity indicator when they derived an ensemble-averaged version of the generalized Burgers equation (GBE) for spherical spreading, which may be written as

$$\frac{\partial}{\partial r}\left[r^2 e^{2\alpha(f)r} S_{pp}(r,f)\right] = -2\pi f r^2 \frac{\beta}{\rho_0 c_0^3} e^{2\alpha(f)r} Q_{p^2p}(r,f). \tag{2}$$

The GBE is a parabolic (one-way) propagation equation that can incorporate nonlinearity, geometric spreading, and atmospheric absorption and dispersion. In Eq. (2), the left-hand side represents the spatial rate of change of the power spectral density, $S_{pp}(r,f)$, that has been corrected for linear processes, namely spherical spreading (r^2) and atmospheric absorption ($e^{2\alpha(f)r}$). If free-field, far-field, linear propagation holds, Eq. (2) approaches zero. Therefore, the right-hand side of Eq. (2) represents a quadratic source term that accounts for the sum/difference frequency generation that occurs during nonlinear propagation. At frequencies where the right-hand side is negative, there is a net energy loss due to nonlinearity (i.e., energy is being transferred to other frequencies). At frequencies where the right-hand side is positive, the energy net gain is positive.

Different forms of nonlinearity indicators have evolved from the right-hand side of Eq. (2), all involving $Q_{p^2p}(r,f)$,[5-10] but in this work the right-hand side of Eq. (2) itself will be used. Physically, the right-hand side of Eq. (2) is the spatial rate of change of the power spectral density, $S_{pp}(r,f)$, due to nonlinearity as a function of frequency and range.

RESULTS

Displayed in Fig. 1 are two cases of propagation from 10-75 jet nozzle diameters (D_j). The measurements were made with a microphone array having its origin located 4 D_j downstream of the nozzle exit. For the subsonic jet measurements, the 4 D_j array origin is very close to the dominant noise source region based on phased-array measurements for a Mach-0.9 unheated jet.[11] First, as a baseline case, is a Mach 0.85 unheated subsonic jet along a propagation angle of 90° relative to the upstream direction. Figure 1a shows the measured spectra and Fig. 1c shows the collapse of the measured spectra at 75 D_j after spherical spreading and atmospheric absorption have been incorporated. The agreement suggests linear, free- and far-field propagation. On the other hand, Figs. 1b and 1d show the Mach 2.0 jet data along 145°, which is the maximum far-field radiation angle. There is a shift downward in peak frequency along this measurement radial between 10 and 60 D_j. At high frequencies, the spectral slope (approximately -20 dB/decade) is maintained, unlike the 90° Mach 0.85 jet result. The attempt to collapse the spectra via a linear, far-field model in Fig. 1d reveals that there is more energy at low and high frequencies measured at 75 D_j than predicted.

Figure 2 shows the right-hand side of Eq. (2) plotted for Mach 0.85 and Mach 2.0 jet data. Figure 2a demonstrates the behavior of the nonlinearity indicator when linear collapse of the Mach 0.85 data is quite good. The noisiness and small magnitude of the curve as a function of frequency suggests no trend in terms of energy transfer. A similar result was seen for the F-18E aircraft at idle.[7] On the other hand, the nonlinearity indictor shows, for 145° between 10 and 75 D_j, that the Mach 2.0 jet is transferring energy upward in the spectrum and that there is little evidence of energy transfer downward in the spectrum. Given the similarity in peak frequencies between the two sets of spectra 75 D_j (Fig. 1a and Fig. 1b), it is also useful to compare magnitudes. Note that a four-order of magnitude change in sound pressure level (~40 dB) has resulted in a change in the quadspectral indicator of *more* than six orders of magnitude. This is in keeping with the Pa^3/Hz units of Q_{p^2p} and an increased tendency toward nonlinear propagation.

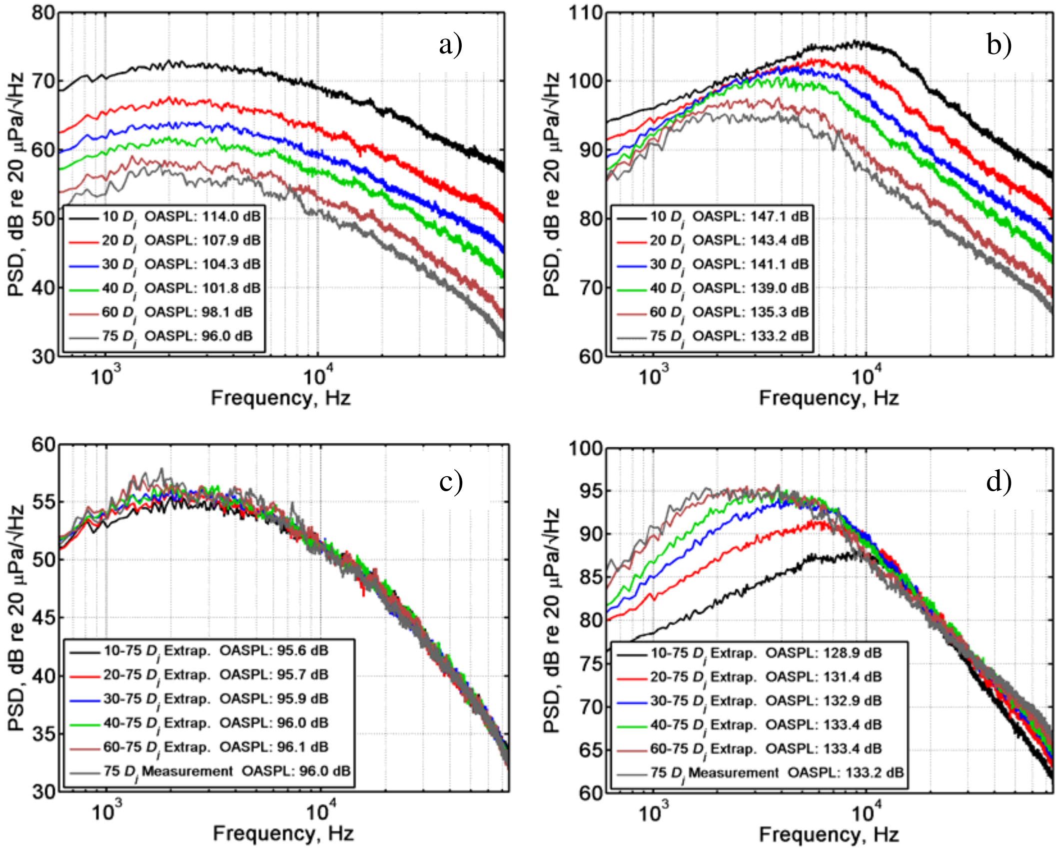

FIGURE 1. Measured spectra at a) 90° for a Mach 0.85 unheated jet and b) 145° for a Mach-2.0 unheated jet. c) Linear collapse for the part a); d) Linear collapse for part b).

Overall, the results show the utility in examining nonlinear propagation using this type of analysis. In the specific case of the Mach-2.0, unheated jet, the excess of high-frequency energy at 75 D_j is due to a nonlinear transfer of energy upward in the spectrum, but the low-frequency excess is caused other effects, possibly by directional radiation from extended sources.

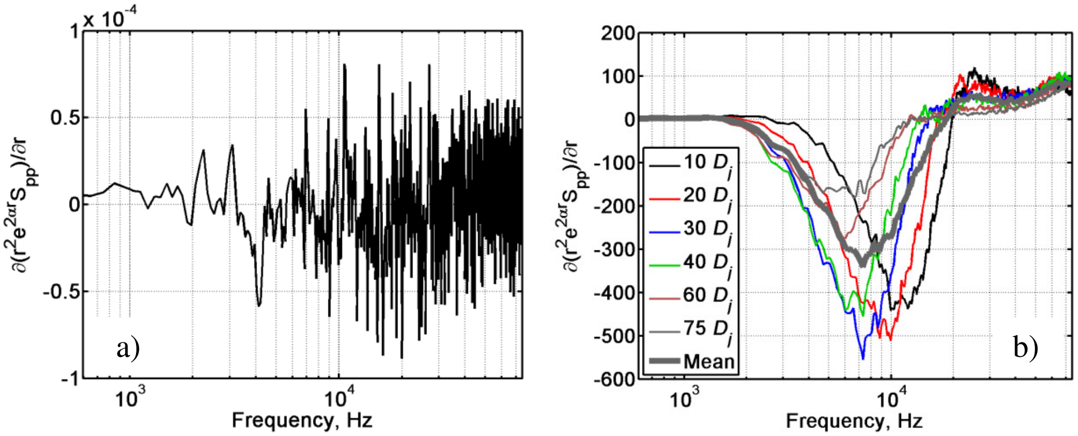

FIGURE 2. Quadspectral indicator for a) Mach 0.85 jet at 90° and 60 D_j. and b) 10-75 D_j, along with the mean, for 145° and the Mach 2.0 jet.

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