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ESTIMATING ACOUSTIC RADIATION USING WAVENUMBER SENSORS

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INTRODUCTION

A number of current problems of interest in active control involve the need to control the acoustic radiation from a vibrating structure. It has been demonstrated that the optimal solution for minimizing the radiation often does not correspond to minimizing the vibration of the structure. As a result, there has been considerable interest in exploiting the radiation mechanisms for a vibrating structure as a means of controlling the acoustic radiation.

Several control schemes have been proposed that are based on a model of the acoustic radiation [1,2,3]. An alternative method of minimizing the radiation is to directly estimate the wavenumber spectrum of the vibrating structure, from which the supersonic (radiating) components of the spectrum can be identified and minimized [4]. This method has the advantage of providing a direct measure of the radiation mechanism, which is not based on a model of the system. As a result, if the parameters of the system change, the method would be sensitive to those changes and capable of tracking the changing radiation conditions.

This paper outlines a method to obtain an estimate of the structural wavenumber spectrum, from which the radiation can be estimated. The approach is based on the use of an array of distributed sensors, which provides the capability of attenuating the effects of spatial aliasing if there is significant energy in the subsonic (nonradiating) components of the wavenumber spectrum.

ACOUSTIC RADIATION FROM STRUCTURES

To control the energy that is radiated from a structure as efficiently as possible, it is important to base the control strategy on the physical mechanisms associated with radiation. The energy that is radiated from the structure can be conveniently isolated from the energy that is not radiated by formulating the problem in the spatial transform (wavenumber) domain. In this paper, the structure will be assumed to be one-dimensional, such that the transverse velocity of the structure is described by $v(x)$. The spatial Fourier transform of this field is the velocity wavenumber transform, and will be designated by $V(k_x)$. The fluid-structure interaction is described by Euler's equation, which couples the transverse vibration of the structure with the acoustic pressure in the fluid according to

$$\rho_f \left. \frac{\partial u(x,z)}{\partial t} \right|_{z=0} = - \left. \frac{\partial p(x,z)}{\partial z} \right|_{z=0} . \tag{1}$$

Here, ρ_f is the fluid density, z is the direction perpendicular to the plane of the structure, and $u(x,z)$ is the acoustic particle velocity, which is equal to the transverse velocity of the structure at the surface of the structure. For far-field radiation, the pressure field can be expressed as $p = P_o e^{j(\omega t \pm k_{fx} x - k_{fz} z)}$, where k_{fx} and k_{fz} are the components of the acoustic wavenumber in the x- and z-directions. Transforming Euler's equation, with the assumed form for the pressure, leads to

$$P(k_x,0) = \frac{\omega \rho_f}{k_{fz}} V(k_x) . \tag{2}$$

The acoustic power (per unit width) radiated from the structure can be obtained from the integral of the normal acoustic intensity over the surface of the structure, given as

$$\Pi = \int_0^L \frac{1}{2} Re\left\{ p(x,0)\, u^*(x,0) \right\} dx , \tag{3}$$

where L designates the length of the structure, $Re\{\ \}$ designates the real part of the argument, and * designates the complex conjugate operator. The acoustic pressure and particle velocity can be obtained as the inverse transforms of $P(k_x,0)$ and $V(k_x)$, which after some simplification leads to the standard result

$$\Pi = \frac{\omega \rho_f}{4\pi} \int_{-k_f}^{k_f} \frac{|V(k_x)|^2}{\sqrt{k_f^2 - k_x^2}} dk_x , \tag{4}$$

where k_f is the acoustic wavenumber.

Eq. 4 indicates that only structural wavenumbers that satisfy the condition $|k_x| \le k_f$ radiate energy to the acoustic far-field. This radiating portion of the structural wavenumber spectrum is referred to as the supersonic wavenumber spectrum. Thus, a sensing scheme that is capable of providing an estimate of the supersonic wavenumber spectrum would be desirable for a control system designed to minimize the far-field acoustic radiation.

There have been several control approaches proposed over the last several years that have focused on sensing and minimizing the supersonic wavenumber spectrum, either explicitly or implicitly. However, most of these approaches have minimized the supersonic spectrum based on information from an assumed model, exceptions to this being the work of Maillard and Fuller [5] and Sommerfeldt [4]. If the physical system changes over time, an estimate of the radiated acoustic power that is based on an assumed model may not be correct. The alternative approach is to directly estimate the wavenumber spectrum from structural measurements, which is the approach adopted in this paper.

According to the Nyquist criterion, a minimum of two spatial samples per wavelength are required for the shortest wavelength (highest wavenumber) of interest. For a given acoustic wavenumber, this will dictate the minimum number of sensors that can be used to estimate the supersonic wavenumber spectrum. However, if there is significant energy in the subsonic structural wavenumbers, as there often is, aliasing will occur unless a lowpass wavenumber filter can be implemented to attenuate the subsonic wavenumber components. It is possible to accomplish this lowpass wavenumber filtering by using distributed sensors. For this paper, PVDF sensors were assumed, due to their capability of being easily shaped to provide desired filter characteristics. These sensors provide an estimate of the induced strain in the structure. Since the strain is directly related to the transverse displacement, and hence the transverse velocity, one can obtain an estimate of the radiation in a form similar to Eq. 4. By shaping the sensors properly, it is possible to estimate the supersonic wavenumber spectrum with a smaller number of sensors than would be required for point sensors.

MODEL CONFIGURATION

To determine the effectiveness of this approach in estimating acoustic radiation, a clamped-clamped aluminum beam was investigated. The beam was characterized by length 0.914 m, width 0.051 m, and thickness 0.006 m. The resonance frequencies associated with this beam were calculated and the first five of these are shown in Table 1. The decision was made to design the distributed sensors to be able to estimate the supersonic wavenumber spectrum for the first five resonances, which indicates that the shaped sensors should be designed to operate as lowpass wavenumber filters with a cutoff wavenumber of approximately 10 m^{-1}, assuming radiation into air.

For a perfect lowpass wavenumber filter, the PVDF sensors should be infinite in length and shaped according to $\sin(k_c x)/(k_c x)$, where k_c is the cutoff wavenumber. Since the filters must be finite in length, the response of the sensors will roll off in wavenumber, and will have sidelobe responses. After some investigation, it was decided to use a Hamming window multiplying the $\sin(k_c x)/(k_c x)$ shape to minimize the sidelobe effects. In addition, a cutoff wavenumber of 6 m^{-1} was chosen to provide a compromise between low attenuation in the desired passband, and high attenuation in the desired stopband. The lowpass filter characteristics were calculated for various sensor lengths, and are shown in Fig. 1. Based on these results, the length of the PVDF sensors was

Table I. Resonance frequencies for clamped-clamped beam.

Mode	1	2	3	4	5
Freq. (Hz) (Theory)	40.1	110.5	216.5	357.9	534.7

chosen to be 0.6 m, and a total of six sensors were used, as shown in Fig. 2. The spacing between the center points of the sensors is 0.2285 m. For the results presented here, the beam was excited at one of its resonance frequencies by a point force located at x = 0.64 m. The time domain signals from the PVDF sensors were then Fourier transformed into the frequency domain, after which the frequency domain signals at the excitation frequency were spatially Fourier transformed into the wavenumber domain.

NUMERICAL RESULTS

To further examine the approach outlined here, the wavenumber transform of the strain field was determined using three different methods. For each of the five resonance frequencies of interest, the exact wavenumber transform was calculated for the given geometry. In addition, the wavenumber transform was calculated using the six discrete signals obtained from the shaped PVDF sensors. Finally, for comparison, the wavenumber transform was calculated using four point measurements at the locations x = 0.114, 0.343, 0.571, 0.8 m, such as one might obtain from discrete accelerometers. Two representative results, for modes 3 and 4, are shown in Fig. 3 and Fig. 4, respectively. The vertical line at a value

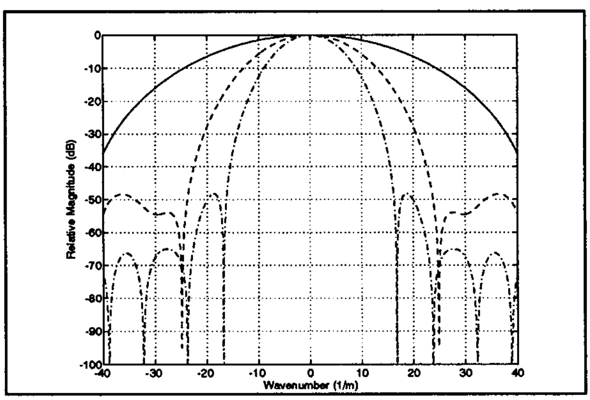

Figure 1. Wavenumber response of shaped sensors. Sensor lengths shown are: ______ 0.3 m; _ _ _ 0.6 m; _._._. 1.0 m;

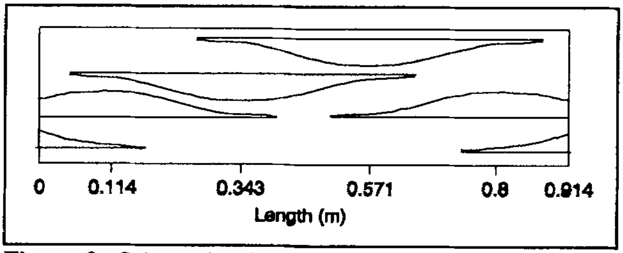

Figure 2. Schematic showing the layout of the shaped sensors on the clamped-clamped beam.

of k = 4 m^{-1} in Fig. 3 and at k = 6.6 m^{-1} in Fig. 4 indicate the value of the acoustic wavenumber.

For mode 3 (Fig. 3), it can be seen that the agreement between the exact spectrum and the spectrum from the shaped sensors is excellent over the supersonic region of the spectrum. At higher wavenumbers, there is significant discrepancy, since the shaped sensors have been designed to suppress this region. In addition, the wavenumber spectrum obtained using point sensors demonstrates noticeable aliasing errors, which would make it difficult to obtain a reasonable estimate of the radiated energy.

For mode 4 (Fig. 4), the agreement between the exact spectrum and the shaped sensor spectrum is again quite good. There is some discrepancy near the acoustic wavenumber, and at very low wavenumbers. For this mode, the acoustic wavenumber is near the design cutoff wavenumber, resulting in a slightly degraded estimate. Again one can see significant aliasing errors if point sensors are used, since they do not suppress any of the subsonic wavenumber components. The results for mode 5 are similar in nature, with the estimated supersonic spectrum being a little more degraded than for mode 4, since the acoustic wavenumber increases further.

CONCLUSIONS

The acoustic energy radiated from a structure can be estimated in a straightforward manner from a knowledge of the supersonic wavenumber spectrum. A method has been outlined that allows one to estimate the supersonic wavenumber spectrum using an array of shaped sensors. Using shaped sensors makes it possible to significantly reduce the number of sensors required for the array, since the subsonic wavenumber components will be attenuated by the sensors. The numerical results presented here indicate that it should be possible to obtain a good estimate of the wavenumber spectrum over the design region of the sensors. Work is currently in progress to mount shaped PVDF sensors on the physical beam and to verify these numerical results.

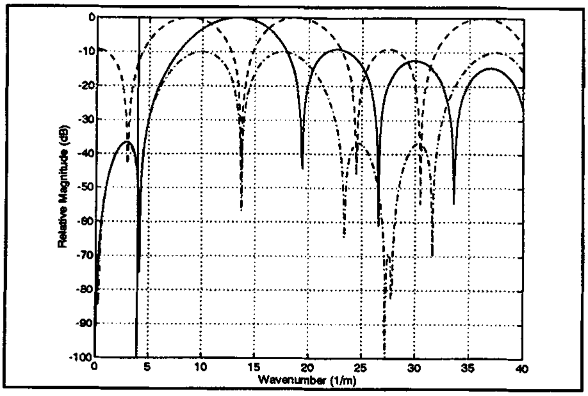

Figure 3. Wavenumber transform of beam excited in its third mode: ______ analytic transform; _ _ _ point sensors; _._._. distributed sensors.

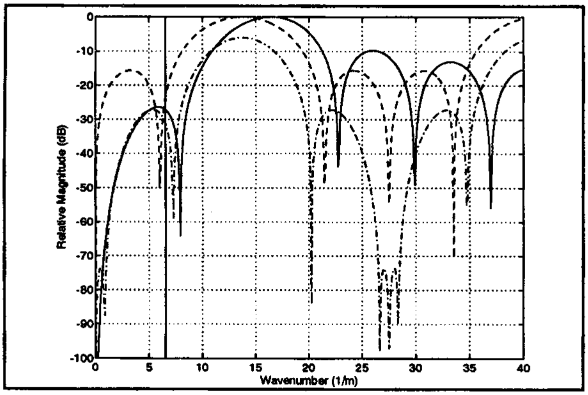

Figure 4. Wavenumber spectrum of beam excited in its fourth mode: ______ analytic transform; _ _ _ point sensors; _._._. distributed sensors.

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