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2pSA10. Acoustic radiation mode shapes for control of plates and shells

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During the advent of active structural acoustic control, attempts were made to target and control structural vibration mode shapes to reduce radiated sound power. In the late eighties and early nineties work on acoustic radiation mode shapes developed an alternative way to target structural acoustic radiation. By attempting to control the radiation mode shapes, contributing structural modes could be more easily targeted. Radiation mode shapes have been examined previously for rectangular plates. The method has been extended to demonstrate radiation mode shapes of circular plates and cylindrical shells. Certain spatial derivatives of plate vibration have been found to be highly correlated with the most efficiently radiating radiation mode shapes at low frequencies. A weighted sum of these spatial derivatives is proposed as a new, generalized control metric.

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INTRODUCTION

Active noise control has been an important problem in acoustics with work on it dating back almost eighty years. About twenty years ago an alternative form of active noise control, termed active structural acoustic control (ASAC), was developed, in which attempts to control radiating structural modes were made in order to reduce the sound radiated by these modes into the sound field. The earliest methods attempted to minimize a quantity such as squared pressure at a microphone (Fuller, 1991) in the sound field by applying a control force to the vibrating structure. This provided for local sound control at the microphone as with other active noise control applications. It was quickly discovered, however, that measuring and attempting to reduce a quantity on the vibrating structure related to the radiation of sound could just as easily be implemented while providing for global sound reduction in the sound field. An early example of this is the measurement of volume velocity as an objective function for ASAC (Elliott, 2002). Measuring a quantity on the structure provided for global control while moving the measurement device to the boundary of the sound field. This was significant because moving the measurement device to the boundary allowed for the potential application of active structural acoustic control to structures in areas where measurement devices in the sound field were not feasible.

About this same time radiation mode shape formulations were developed (Elliott, 1993). It was observed that while all structural modes vibrate, not all structural modes are efficient radiators of sound power. By treating a plate or a shell as a series of elementary radiators an alternative to the Rayleigh integral could be developed for the calculation of sound power. This is given as $W = \frac{1}{2}v^H\mathbf{R}v$ where W is the radiated power, v is a vector containing the average velocities of each elemental radiator and $\mathbf{R}$ is known as the radiation resistance matrix, which takes into account the interactions between the elemental radiators. Radiation mode shapes are derived as the eigenvectors of the radiation resistance matrix. The radiation resistance matrix is shown in Eq. (1) (Fahy, 2007).

$$[R] = \frac{\omega^2\rho_0}{4\pi c}\begin{bmatrix} A_{e1}^2 & \frac{A_{e1}A_{e2}sin(kR_{12})}{kR_{12}} & \cdots & \frac{A_{e1}A_{ej}sin(kR_{1j})}{sin(kR_{1j})} \\ \frac{A_{e2}A_{e1}sin(kR_{21})}{kR_{21}} & A_{e2}^2 & & \vdots \\ \vdots & & \ddots & \\ \frac{A_{ej}A_{e1}sin(kR_{j1})}{kR_{j1}} & \cdots & & A_{ej}^2 \end{bmatrix} \tag{1}$$

The radiation mode shapes are a mathematical abstraction and don't represent physical vibration modes. However, they lump together efficiently vibrating structural mode shapes. Thus, by developing an objective function which targets the radiation mode shapes of a structure, structural vibrations contributing to sound radiation can be targeted directly.

In this paper radiation mode shapes for two geometries, circular plates and cylindrical shells, are discussed and objective functions used to target the most efficient radiation modes are suggested. The examination of circular plates is mainly academic, as these do not occur frequently in structures. Cylindrical shells, however, occur often in man-made structures such as pipes, ducts, aircraft fuselages, rockets, submarine prssure hulls, electric motors, and generators (Farshidianfar, 2011). Much research has been done on the structural modes of cylindrical shells (Soedel, 2004), but little has been published on their radiation modes, therefore it is an interesting and important problem to address for ASAC. In both cases the circular plates and cylindrical shell radiation mode shapes were found to be comparable to the radiation mode shapes of a rectangular plate. Some interesting results with regards to circular plates concerning this are discussed.

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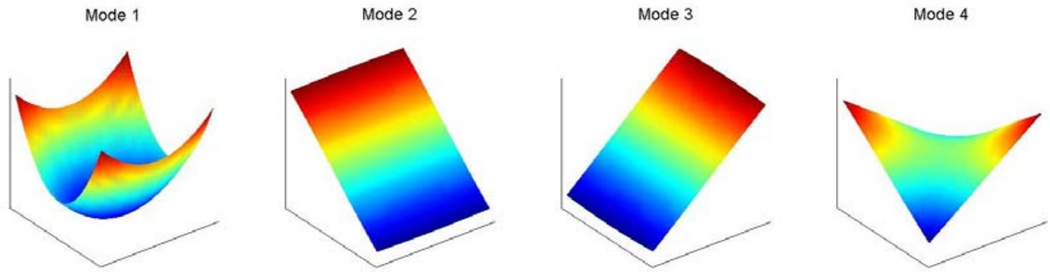

FIGURE 1: Rectangular plate radiation mode shapes

RECTANGULAR PLATE RADIATION MODE SHAPES

Using the formulation given in Eq. (1) the radiation mode shapes for a rectangular plate can be easily found. The first four radiation modes at low frequencies are shown in Fig. 1. As excitation frequencies increase these mode shapes become more exaggerated and their radiation efficiencies decrease. It was found previously that ASAC on a rectangular plate could be done effectively, without regards to sensor placement, by using a sum of weighted spatial derivatives, termed composite velocity $\left(V_{comp}^2\right)$ (Fisher, 2012). It was observed that each of the spatial derivatives was similar to one of the radiation modes shown. A similar observation can be made for circular plates.

CIRCULAR PLATE

The circular plate was assumed to be placed in an infinite baffle. Also, clamped boundary conditions were used and harmonic excitation by a point force was assumed. The displacement of a circular plate under these conditions is given by (Morse, 1968)

$$w(r,\theta) = \sum_q \frac{F_q}{\pi a^2} \sum_{m,n}^{\infty} \frac{\psi_{m,n}(r,\theta)\psi_{m,n}(r_q,\theta_q)}{\Lambda_{mn}(\gamma_{mn}^4 - \gamma^4)} \tag{2}$$

where $\psi_{mn}()$ are the shape functions, q indexes the applied forces, and the summation over m,n sums over the modes beginning at the lowest natural frequency.

Analytical Radiation Mode Shapes

The formulation for the radiation resistance matrix, in Eq. (1), from which the radiation mode shapes are derived as eigenvectors, is well known. One of the difficulties with the circular plate is determining the shape of the elements. It is tempting to break up the circle in circular coordinates using wedge or arc shaped elements. However, one of the requirements in using elementary radiators to calculate sound power is that each radiator has approximately the same area. By using elements as just described there is a large difference in area between the innermost and outermost elements. The radiation mode shapes are distorted by this disparity and the calculation of radiated sound power will be innacurate. Thus for the circular plate square elements were used. As long as these are sufficiently small enough (as a rule of thumb ~ 1/6 of the smallest wavelength of interest) this is not a concern.

The first four radiation mode shapes for a circular plate (of radius 0.1 m) are shown in Fig. 2 and are compared to the spatial derivative terms w^2, $(\partial w/\partial x)^2$, $(\partial w/\partial y)^2$, and $\left(\partial^2 w/\partial x \partial y\right)^2$ used in V_{comp}^2 (found using Eq. (2)). The first radiation mode is similar to transverse displacement,

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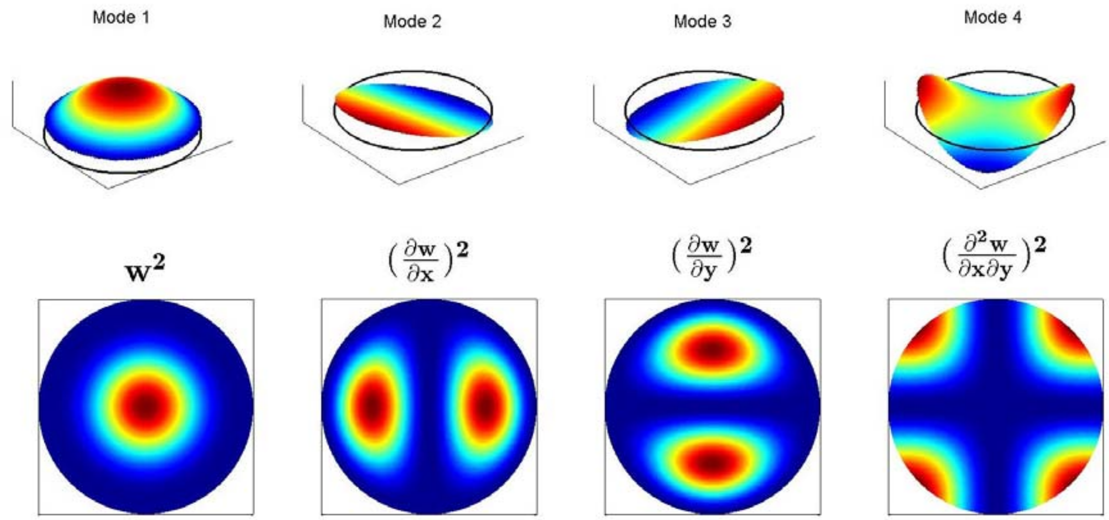

FIGURE 2: The first four radiation mode shapes of a circular plate and the four spatial terms which they relate to.

the second to a rocking motion in x, the third to a rocking motion in y, and the fourth to a twisting motion. This same phenomenon was observed with the rectangular plate, and therefore suggested that V_{comp}^2 would be a potential objective function in the control of circular plates. Volume velocity is similar to the first mode, and was therefore also chosen as an objective function to be tested.

Circular Plate Simulated Control

Control was simulated using volume velocity and V_{comp}^2, as stated above. The simulated control is shown in Fig. 3. V_{comp}^2 attempts to provide a more uniform objective function, therefore being less sensitive to sensor location. It was reported to be effective on rectangular plates because of the correlation between the spatial terms and the radiation mode shapes, but was found to be relatively ineffective on the circular plate. Using this objective function control was relatively good at the first, fourth, and ninth modes, but poor along the rest of the spectrum. Volume velocity, on the other hand was found to give excellent control over the frequency range investigated (1 to 165 Hz), with the exception of the sixth and thirteenth modes.

There are several reasons why volume velocity performs well as an objective function and V_{comp}^2 does not. On the circular plate there are multiple modes that radiate extremely poorly, or not at all. For example, the second and third modes, shown in Fig. 4 do not appear to radiate at all. Upon examination, because of intercellular cancellation these modes have zero volume velocity. This was found to be typical of other modes with poor or nonexistent radiation efficiencies. Most modes that did radiate, such as modes one and four, have a net volume velocity.

A relatively large amount of control was predicted for both volume velocity and V_{comp}^2 at the first mode, because both of these objective functions take into account the displacement at the first mode. At later modes, however, volume velocity significantly outperforms V_{comp}^2. The reason for the good performance of volume velocity as an objective function is that the modes that don't radiate have no net volume velocity, so they aren't targeted, while the modes that do radiate usually have a net volume velocity so they are targeted. To support this conclusion examine mode six, which volume velocity does poorly at controlling. In Fig. 4 it can be seen that the net volume velocity of this mode is zero, but Fig. 3 shows that this mode radiates. Thus because this mode has zero net volume velocity, yet still radiates, volume velocity does not

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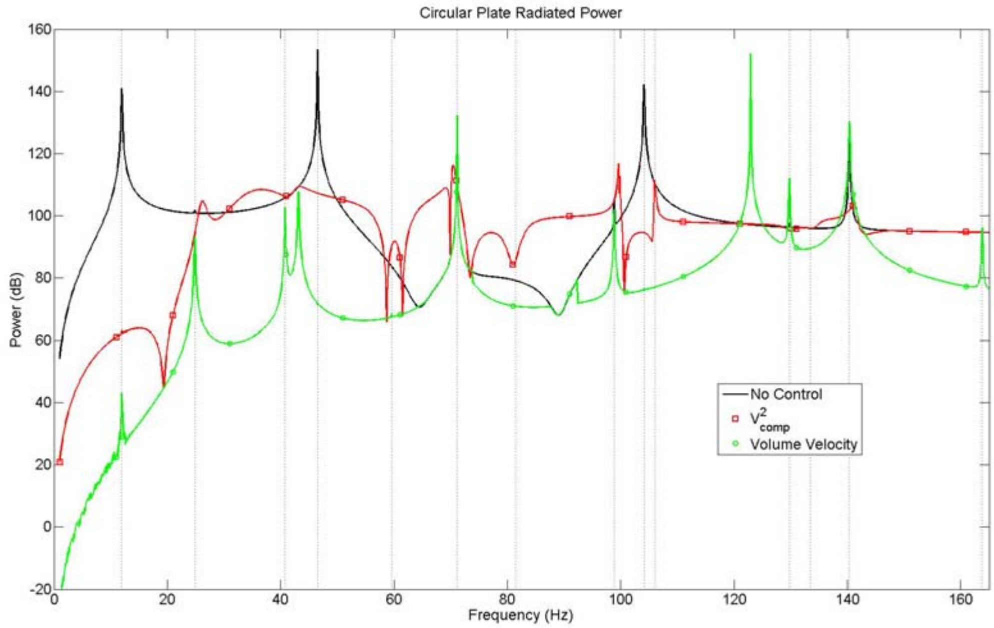

FIGURE 3: Simulated radiated power of the circular plate using volume velocity and V^2_{comp} as objective functions (natural frequencies indicated by dashed lines).

provide attenuation. This is also the case for the thirteenth mode.

The reason that V^2_{comp} provides poor control over most of the frequency range examined is due to the spatial terms. On the rectangular plate these contributed to control and provided for insensitivity to sensor placement. However, on the circular plate the spatial derivative terms are most similar to the second and third modes, which don't radiate. Thus while attempting to control these modes for structural vibration no sound attenuation is achieved. On or near modes where there is net volume velocity, such as the first, fourth, and ninth modes the displacement term of V^2_{comp} provides for some control, but away from these modes control is lost. Therefore the spatial derivatives detract from overall ability of the objective V^2_{comp} to control the plate.

From the preceding analysis it can be seen that volume velocity is generally well fitted for the control of circular plates, whereas V^2_{comp} is not. It can therefore be concluded that because volume velocity targets both radiation modes and radiating structural modes it is an effective objective function for ASAC on a circular plate, and that although V^2_{comp} targets radiation modes it also targets structural modes which don't radiate, therefore it is ineffective for ASAC on a circular plate. It should be noted, however, that while volume velocity has been shown to be an effective objective function on circular plates it requires a global measurement, needing a large number of sensors for an accurate estimate, and is therefore difficult to implement in practice.

CYLINDRICAL SHELLS

For the cylindrical shell simply supported boundary conditions were assumed. The displacement of a cylindrical shell is given by Eq. (3) (Soedel, 2004) where U_x is the displacement along the long axis of the cylinder, U_θ is the displacement along the

$$\begin{bmatrix} U_x \\ U_\theta \\ U_3 \end{bmatrix}_i = C_i \begin{bmatrix} \frac{A_i}{C_i} \cos \frac{m\pi x}{L} \cos n(\theta-\phi) \\ \frac{B_i}{C_i} \sin \frac{m\pi x}{L} \sin n(\theta-\phi) \\ \sin \frac{m\pi x}{L} \cos n(\theta-\phi) \end{bmatrix} \tag{3}$$

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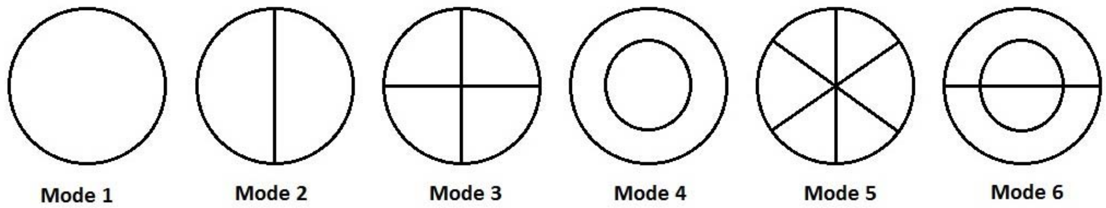

FIGURE 4: The first 6 structural modes of the circular plate

circumferential direction and U_3 is the displacement normal to the surface of the cylinder. The different structural mode shapes are determined by the values of m and n, where m is the axial mode number and n is the circumferential mode number. One should note that natural frequencies of the system $\omega_{m,n}$ are embedded inside the amplitudes of the complex coefficients A_i, B_i and C_i. These amplitudes are also functions of strains and stresses of the system which depend on system properties. The subindex i stands for 3 different natural frequencies avilable for each set of m and n mode numbers.

Different mode shapes of the system have been verified by computer simulation for a thin shell, and are shown in Fig. 5. Here, scaling has been chosen to magnify the displacement of the system. These were compared with the radiation mode shapes of the cylindrical shell. The radiation mode shapes of the system were determined by dividing the cylindrical surface into elementary radiators to construct the radiation resistance matrix and find its eigenvectors (or radiation mode shapes).

In the process of building the radiation resistance matrix, one needs to pay attention to how to calculate the correct distance between elementary radiators. Every radiator interacts with every other radiator directly through the center of the shell as well as along an arc across the outside surface. It was determined that the most accurate estimation of sound power was calculated by using the arc distance along the surface between the elementary radiators. The first four radiation mode shapes for low frequencies can be seen in Fig. 6.

An important connection has been observed beween the structural modes and radiation modes. Comparing the structural modes to the radiation modes gives an idea of which structural modes contribute most to the sound radiation of the cylindrical shell. As with the circular plate, this helps in choosing an objective function for ASAC which will target efficiently radiating structural modes specifically. This knowledge makes it possible to effectively develop or choose an objective function which will require fewer sensors and be less sensitive to sensor

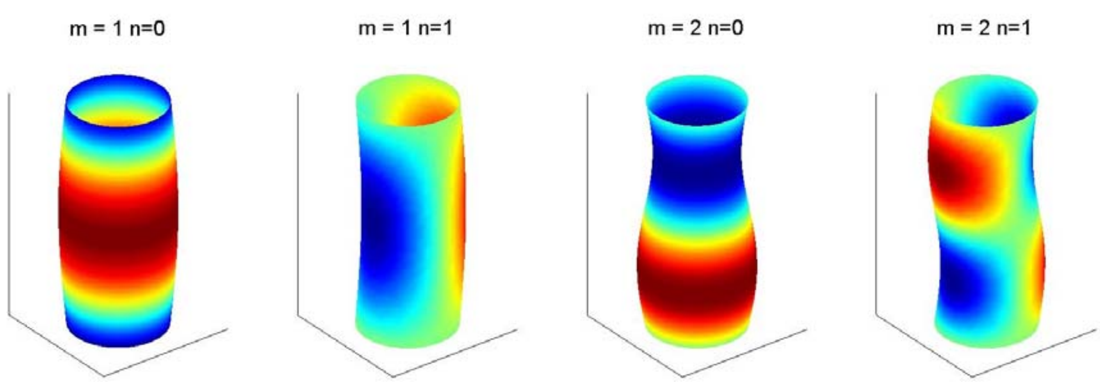

FIGURE 5: First four modes of a simply supported cylindrical shell

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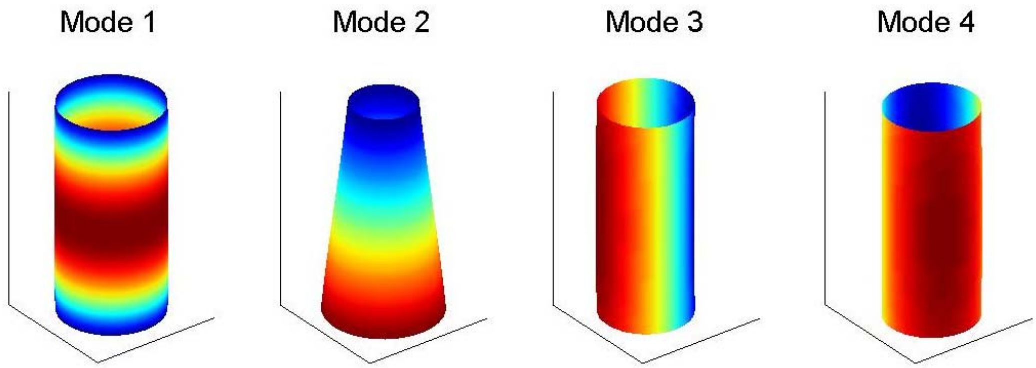

FIGURE 6: First four radiation mode shapes of the cylindrical shell

location while still providing a large amount of sound attenuation.

Comparison to the Rectangular Plate

When the cylindrical radiation mode shapes are compared to the rectangular radiation mode shapes some similarities can be observed. The first radiation mode of the cylinder is like the first radiation mode of the rectangular plate if the cylinder is unwrapped. Likewise the second, third and fourth modes of the cylinder are all similar to the rocking modes (modes two and three) of the rectangular plate. Also, although not shown here, there are radiation modes on the cylinder which can be compared to a cylindrically wrapped version of the twisting mode (mode 4).

These comparisons to the rectangular plate suggest that V_{comp}^2 is a potential objective function for the cylindrical shell because it targeted similar radiation modes on the rectangular plate. Also, due to the greater similarity in geometry between the cylindrical shell and the rectangular plate, the problems associated with control using V_{comp}^2 on the circular plate may not appear. Specifically, the circular plate radiated most efficiently when the net volume velocity did not equal zero. However, on the rectangular plate, and it is anticipated on the cylindricical shell, radiation still occurred even when the net volume velocity was zero due to incomplete intercellular cancellation.

To continue work on determining a more effective method for ASAC on cylindrical shells potential objective functions need to be studied more rigorously. Some potential objective functions to be considered include V_{comp}^2, volume velocity, and energy density. Control of these will be simulated, and at some future date tested experimentally.

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