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Surface conductance of a copper wire in a fluid at high pressure

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The three-dimensional flow of heat in a wire carrying a current and immersed in a liquid is solved in detail. Using this exact result the surface conductance of copper in petroleum ether has been measured as a function of pressure to 40 kbars. The measured surface conductance for copper in a fluid is very small, justifying approximations which yield results that are in agreement with a simplified one-dimensional heat-flow problem. Surprisingly, even at 40 kbars pressure a very large fraction of the joule heating within a wire with a length-to-diameter ratio of ~100 is dissipated through the ends of the wire rather than to the surrounding liquid.

I. INTRODUCTION

Careful null-type thermodynamic measurements depend upon a knowledge of heat flow to the sample through the electrical leads. We considered whether these problems are minimized by making the leads short and fat to reduce joule heating in the leads or making them long and thin to dissipate the heat through the surface of the wire before it enters the sample. This question could not be answered without a knowledge of the heat flow in and through the surfaces of the wire. The approximation of a wire of negligible cross section, which has been discussed in the literature,[1] was inappropriate for this study because we wished to know the results for all ratios of length to diameter.

A wire carrying a current will be heated above its ambient temperature due to joule heat liberated throughout the volume of the wire. The process of heat transfer through the cylindrical surface of the wire into surroundings of uniform temperature can be analyzed by a boundary condition sometimes called Newton's law of cooling.[2] This condition assumes that the flux of heat through the surface at each point along the wire is proportional to the temperature discontinuity between the wire's surface and the contacting medium. The proportionality constant is called the surface conductance. The heat flow in a circular wire of finite length carrying a constant current can be solved with the surface conductance as a parameter. But the size of this parameter is not easily determined. If the surface is perfectly insulating, the surface conductance is zero; but on the other hand, if the surrounding medium is efficient enough at removing heat from the wire, such as to keep the wire's surface at the temperature of its surroundings, then the surface conductance is infinite. The value of this parameter thus depends upon the nature of the wire and the surrounding fluid, and its value is not available in handbooks. A method of measuring surface conductance is presented here along with some data on such a measurement for a copper wire in petroleum ether at various pressures to 40 kbars.

II. ANALYSIS

Carslaw and Jaeger[1] discuss a simple approximation in which the wire is assumed very long compared to its diameter and radial flow of heat within the wire is neglected; i.e., one assumes a constant temperature across any cross section of the wire. The steady-state problem becomes a one-dimensional differential equation readily solvable in terms of hyperbolic functions. Since we are also interested in the case of large diameter wires, an exact three-dimensional solution is required. This solution can then be simplified where the situation warrants. As will be pointed out, an approximate solution is adequate for the geometry used in this research to measure the surface conductance.

We consider a circular cylindrical wire of length l and radius a and use the steady-state heat equation[2] in cylindrical coordinates for this cylindrically symmetric problem. Take the z axis to be the axis of the wire and r the radial coordinate. We let $T(r,z)$ be the temperature and $j^2\rho$ be the rate of heat generated per unit volume in the wire, where j is the electric current density and ρ the electrical resistivity. The heat equation is then

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = -\frac{j^2\rho}{\kappa}, \tag{1}$$

where κ is the thermal conductivity of the wire material.

For boundary conditions we assume a heat sink of temperature T_0 at the surfaces $z=0$ and $z=l$ and the Newton boundary condition at the cylindrical surface $r=a$ with surface conductance H. No generality is lost by letting $T_0=0$, as then the temperature in the problem is relative to T_0. A dimensionless parameter is also introduced to simplify the analysis: $h \equiv Ha/\kappa$. The radial boundary condition is then

$$a\left(\frac{\partial T}{\partial r}\right)_{r=a} = -hT(a,z). \tag{2}$$

A problem very similar to this one is solved by Green's function techniques in Morse and Feshbach.[3] The only difference is that the boundary condition for their problem is $T=0$ on all bounding surfaces; $z=0$, $z=l$, and $r=a$. Their solution gives for $T(r,z)$, with A being a constant of proportionality,

$$T(r,z) = A\sum_{s,n}\frac{j^2\rho l^2 J_0(x_s r/a)G(n,x_s)}{x_s J_1(x_s)}, \tag{3}$$

where x_s are the roots of the equation $J_0(x_s)=0$, $J_n(x)$ is the Bessel function of order n, and

$$G(n,x_s) \equiv \frac{\sin[(2n+1)\pi z/l]}{(2n+1)[(2n+1)^2+(x_s l/a\pi)^2]}.$$

Note that with modification this result can also be the solution for the boundary conditions of the problem considered here, as is seen by substituting Eq. (3) into (2) using the relation[4] $J'_0(x) = -J_1(x)$ to obtain

$$\sum_{s,n} G(n,x_s) = \sum_{s,n} \frac{hJ_0(x_s)G(n,x_s)}{x_s J_1(x_s)}.$$

This equation is satisfied term by term if for every s the x_s satisfy

$$hJ_0(x_s) = x_s J_1(x_s). \tag{4}$$

Thus it is only required that the sum over s in Eq. (3) be over all x_s satisfying Eq. (4). To find the constant A, one uses the physical requirement that the heat passing through the surfaces of the wire must equal the heat generated within the volume of the wire. This is simple to calculate in the limit where $l \to 0$, for then the heat passing through the surface at $z = l$ must be half the total heat generated in the wire. The flux of heat through this surface is $j_Q(l,r) \equiv -\kappa(\partial T/\partial z)_{z=l}$, and the total heat through this surface is

$$Q(l) = \int_0^a j_Q(l,r) 2\pi r\, dr.$$

Performing these operations using the integral over Bessel functions from Ref. 4 and the sum[5]

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2 + (x_s l/a\pi)^2} = \frac{a\pi^2 \tanh(x_s l/2a)}{4x_s l},$$

one obtains

$$Q(l) = \tfrac{1}{2} A j^2 \rho \pi^4 a^3 \kappa \sum_s \tanh(x_s l/2a)/x_s^3. \tag{5}$$

In the limit as $l \to 0$, and with $I = j\pi a^2$ and $R = \rho l/\pi a^2$, Eq. (5) gives

$$\frac{Aj^2\rho\pi^4 a^2 l\kappa}{4} \sum_s x_s^{-2} = \frac{1}{2} I^2 R = \frac{1}{2} j^2 \pi a^2 \rho l,$$

from which $A = 2/(\pi^3 \kappa \Sigma x_s^{-2})$. The complete solution for the temperature is now

$$T(r,z) = \frac{2j^2\rho l^2}{\pi^3\kappa} \sum_s \frac{J_0(x_s r/a)}{x_s J_1(x_s)} \sum_{n=0}^{\infty} \frac{\sin([2n+1]\pi z/l)}{(2n+1)[(2n+1)^2 + (x_s l/a\pi)^2]} \Big/ \sum_{s'} x_{s'}^{-2}, \tag{6}$$

where the x_x satisfy Eq. (4).

The average temperature of the wire is

$$\langle T \rangle = \frac{1}{\pi a^2 l} \int_V T(r,z) 2\pi r\, dr\, dz,$$

which after the integration can be expressed as

$$\langle T \rangle = \frac{\rho I^2}{\pi^2 a^2 \kappa} \frac{\Sigma_s [x_s^{-4} - (2a x_s^{-5}/l)\tanh(x_s l/2a)]}{\Sigma_s x_s^{-2}} = \frac{\rho I^2 f(h)}{\pi^2 a^2 \kappa}. \tag{7}$$

The function $f(h)$ is defined by Eq. (7). To obtain Eq. (7) we have made use of the sum[5]

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2[(2n+1)^2 + (x_s l/a\pi)^2]} = \frac{a^2\pi^4}{8x_s^2 l^2}\left(1 - \frac{2a}{x_s l}\tanh\frac{x_s l}{2a}\right).$$

The resistance of a wire over a small range of temperature is given by $R(T) = R_0(1 + \alpha T)$. From the linearity of this relation one obtains $\langle R \rangle = R_0(1 + \alpha\langle T \rangle)$. Substituting Eq. (7) into this relation for the resistance of a wire carrying a current, one finds

$$R = R_0\{1 + [\alpha\rho f(h)/\pi^2 a^2 \kappa] I^2\}. \tag{8}$$

Thus a linear dependence of R vs I^2 is predicted. The results of such measurements are shown in Fig. 1. It is seen that the linear relation is accurately demonstrated by the data. By measuring the slopes of these curves and using Eq. (8), an experimental value of $f(h)$ can be determined.

The value of h is found by iteration using a computer. One chooses an initial value of h with which one solves Eq. (4) for the roots x_s, then with these one evaluates the sum in Eq. (7) to obtain $f(h)$. From this a better estimate for h is made and the process repeated until the calculated value of $f(h)$ matches that experimentally measured from the slope of R vs I^2.

The value of h for copper wire, of the diameter used, in petroleum ether was not *a priori* known, but it was observed after the measurement and subsequent calculation to be $< 10^{-3}$. For such small values of h an approximation to Eq. (4) can be used which greatly simplifies the computation. For small h one expects the lowest root of Eq. (4) to be near

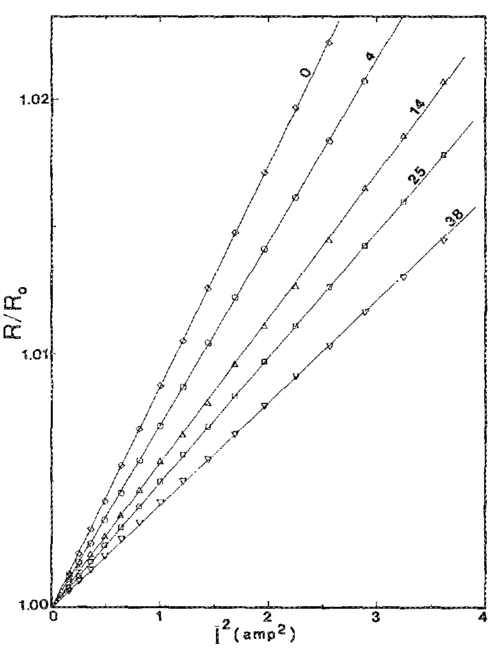

FIG. 1. Resistance of a copper wire vs the square of the measuring current for the wire in petroleum ether at various pressures in kbars indicated on each curve.

zero, so the power-series expansion of the Bessel functions[6] can be used to get the lowest root $x_1^2 \cong 2h - \frac{1}{2}h^2$. For h less than 10^{-3} the values of x_1 are so small that for all the sums in the above equations the first term dominates. Thus only the first term need be considered, and

$$f(h) \cong \frac{1}{2h}\left[1 - \frac{2a}{l\sqrt{2h}}\tanh\left(\frac{\sqrt{2h}\,l}{2a}\right)\right], \tag{9}$$

$$Q(l) \cong \frac{I^2\rho}{\pi a\sqrt{2h}}\tanh\left(\frac{\sqrt{2h}\,l}{2a}\right), \tag{10}$$

and

$$T\left(a,\frac{l}{2}\right) \cong \frac{I^2\rho}{2\pi^2\kappa a^2 h}\left[1 - \mathrm{sech}\left(\frac{\sqrt{2h}\,l}{2a}\right)\right]. \tag{11}$$

The function $f(h)$ has a maximum value of $l^2/12a^2$ in the limit as $h \to 0$. For Eq. (11) the following sum[5] is employed:

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)[(2n+1)^2 + (x_s l/a\pi)^2]} = \frac{\pi^3 a^2}{4x_s^2 l^2}\left[1 - \mathrm{sech}\left(\frac{x_s l}{2a}\right)\right].$$

The error in this approximation for $h \leqslant 0.001$ is less than 0.05% in $f(h)$ and less than 0.01% in $Q(l)$ and $T(a,l/2)$.

The one-dimensional approximation in Carslaw and Jaeger[1] yields

$$T(z) = I^2\rho/2\pi^2\kappa a^2 h\,(1 - \{\sinh(\sqrt{2h}\,z/a) + \sinh[\sqrt{2h}\,(l-z)/a]\}/\sinh(\sqrt{2h}\,l/a),$$

which leads to exactly the same results as Eqs. (9)–(11).

III. EXPERIMENTAL RESULTS

A copper wire of 0.0127 cm diam and of 1.3 cm length was soldered to two 0.05-cm-diam copper leads at each end. This sample was placed in a sealed capsule filled with petroleum ether and put in a pyrophyllite cube similar to the manner discussed by Decker, Jorgensen, and Young.[7] The pressure system is briefly discussed by Vanfleet *et al.*[8] The pressure is only accurate to ± 0.5 kbar, being taken from a load pressure calibration chart made by comparing the load versus the reading on a manganin coil in a similar cell. A current was driven through two of the four leads to the sample by a constant current source and monitored by the voltage drop across a standard resistor in series with the section of copper wire. This voltage was measured by a Keithley 5900 digital voltmeter. The potential drop across the wire was measured on the other leads by an HP 3456A digital voltmeter. These data were fed to a HP9825A computer which continuously calculated the resistance. The resistance at several chosen currents was measured and plotted as a function of I^2. These measurements were repeated at several pressures below 40 kbars. The results of these measurements are shown in Fig. 1. One measurement was also made in air in the same closed container at atmospheric pressure. The ambient temperature was 21 °C for all measurements.

Figure 2 shows the pressure dependence of the resistance of copper to 40 kbars. This is the resistance extrapolated to zero measuring current. The pressure coefficient of electrical resistance, $-0.001\,80$ kbar^{-1}, is in excellent agreement with $-0.001\,78(17)$ kbar^{-1} as reported by Bridgman.[9] Figure 3 shows the value of the surface conductance versus pressure for copper in petroleum ether. The upswing in the results above 30 kbar is due to glassification of the 65–120 °C boiling-point petroleum ether used in the experiment. Figure 4 gives the percentage of the total joule heat that passes out through either end of the wire as a function of pressure and the discontinuity in temperature between the wire and the fluid at the center of the wire with 1 A of current in the wire. The values at higher pressures are all corrected for pressure dependence of the resistance (taken from this work), the dimensions,[10] and the thermal conductivity (from Bridgman.)[11] The variation of α with pressure is negligible, as shown by the absence of a temperature effect on the variation of resistance with pressure.[9] The dominant pressure contribution is the variation of the thermal conductivity with pressure.

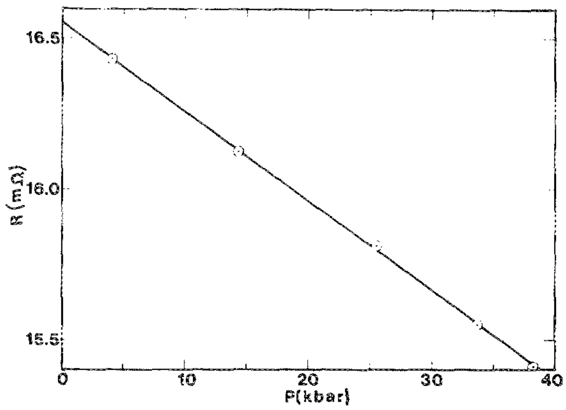

FIG. 2. Resistance of a copper wire vs pressure extrapolated to zero measuring current.

The results in air at atmospheric pressure reveal certain problems with the measurement. The measured slope of R/R_0 vs I^2 exceeds the maximum theoretical value of $\alpha\rho l^2/12a^4\pi^2\kappa$ when using the values from the tables in Aschroft and Mermin[12], namely, $\alpha = 0.0040(1)$ °C^{-1}, $\rho = 1.71(1)$ $\mu\Omega$ cm, $\kappa = 3.83(9)$ W cm^{-1} deg^{-1}. The uncertainties in

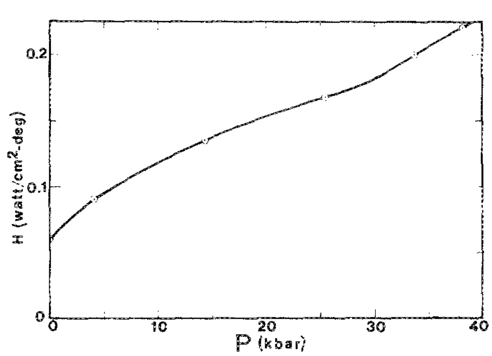

FIG. 3. Surface conductance of a copper wire in petroleum ether as a function of applied hydrostatic pressure. The pressure is hydrostatic below 30 kbars.

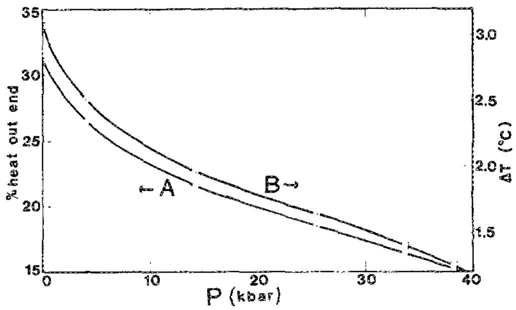

FIG. 4. (A) The percentage of the joule heat that leaves the wire through each end. (B) The temperature discontinuity between the wire and surrounding fluid at the center of the wire with 1 A flowing.

these parameters were taken by comparing to values in other tables. The measured wire length of the section of No. 36 gauge bare copper wire was $l = 1.30(5)$ cm. We assumed a tolerance in the wire radius of about 1% since it is not measureable with good precision. With these values the theoretical maximum slope for R/R_0 vs I^2 is 0.0157(15) while the measured slope in air is 0.0163. The uncertainty in the theoretical maximum slope does overlap the measurement allowing a maximum value of H for copper wire in air at atmospheric pressure to be determined, i.e., $H < 0.0060$ W cm^{-2} deg^{-1}.

IV. CONCLUSIONS

A correct three-dimensional solution of the heat flow in a wire immersed in a liquid and carrying a current has been derived. The linearity of measured resistance versus I^2, as demonstrated in Fig. 1, justifies the use of the Newton boundary conditions. Because the surface conductance between a good metal, such as copper, and a liquid is so small one can use Eqs. (8) and (9), along with the measured resistance of the metal wire as a function of the square of the measuring current, to determine the surface conductance in a straightforward way.

If one used materials which were thermally more compatable, so that the surface conductance was large, or a wire with lower thermal conductivity, or one of larger diameter, then one would have to use the exact equation, Eq. (7), along with these measurements to determine H.

The largest error in the present measurement is the available values of α and κ. Convection and radiation effects are neglected, and one assumes that the fluid acts as a thermal reservoir and maintains the surrounding temperature without developing an internal temperature gradient. To make these assumptions as realistic as possible, the temperature increase of the wire is kept below a few degrees Celsius during the measurement.

The surprising result of this experiment is the fact that even for a 0.0127-cm-diam copper wire with length to diameter ratio of 102, a rather large fraction of the joule heating is conducted out the *ends* of the wire, as shown in Fig. 4. The corresponding value for the copper wire in air in this experiment is $>47\%$ at each end. This explains why it is so easy to heat a wire to glowing by passing a current through it. The temperature at the center of this wire in air with a current of 1 A exceeds the ambient air temperature by 6 °C. This also gives warning that one must take care to limit the current density when measuring the resistance of a wire.

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